YOUR PRACTICE PAPER

APPLICATIONS AND INTERPRETATION

HIGHER LEVEL
FOR IBDP MATHEMATICS

ANSWERS

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- 4 Sets of Practice Papers
- Distributions of Exam Topics
- Exam Format Analysis
- Comprehensive Formula List

SE PRODUCTION LIMITED

AI HL Practice Set 1 Paper 1 Solution

1. (a) The mean ball speed

$= \frac{80+76+100+66+40+116+90+76}{8}$ (A1) for correct formula

$= 80.5 \text{ kmh}^{-1}$ A1

[2]

(b) (i) 78 kmh^{-1} A1

(ii) 21.3 kmh^{-1} A1

(iii) 76 kmh^{-1} A1

[3]

2. (a) $u_{10} = 181$

$\therefore 100 + (10-1)d = 181$ (A1) for correct equation

$9d = 81$

$d = 9$ A1

[2]

(b) 208 A1

[1]

(c) The total number of seats

$= \frac{15}{2}\left[2(100) + (15-1)(9)\right]$ (A1) for substitution

$= 2445$ A1

[2]

3. (a) $\cos A\hat{B}C = \dfrac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$ (M1) for cosine rule

$\cos A\hat{B}C = \dfrac{28^2 + 41^2 - 32^2}{2(28)(41)}$ (A1) for substitution

$\cos A\hat{B}C = 0.6276132404$

$A\hat{B}C = 51.12574956^\circ$

$A\hat{B}C = 51.1^\circ$ A1

[3]

(b) The area of the park

$= \dfrac{1}{2}(AB)(BC)\sin A\hat{B}C$ (M1) for area formula

$= \dfrac{1}{2}(28)(41)\sin 51.12574956^\circ$ (A1) for substitution

$= 446.873514\ \text{m}^2$

$= 447\ \text{m}^2$ A1

[3]

4. (a) (i) The gradient of L

$= -1 \div \dfrac{5-1}{7-5}$ (M1) for valid approach

$= -1 \div 2$

$= -\dfrac{1}{2}$ A1

(ii) The equation of L:

$y - 4 = -\dfrac{1}{2}(x - 4)$ (M1) for substitution

$y = -\dfrac{1}{2}x + 6$ A1

[4]

(b) Kimberly's office is on the boundary separating the Voronoi cells of the restaurant B and the restaurant C, which is equidistant to them. R1

[1]

5. (a) The expected number

$=(13)(0.25)$ (A1) for substitution

$=3.25$ A1

[2]

(b) The variance

$=(13)(0.25)(1-0.25)$ (A1) for substitution

$=2.4375$ A1

[2]

(c) The required probability

$=\binom{13}{8}(0.25)^8(1-0.25)^{13-8}$ (A1) for substitution

$=0.0046602041$

$=0.00466$ A1

[2]

6. (a) (i) $y=20-4x$ A1

(ii) $0<x<5$ A1

[2]

(b) $V=(4x)(2x)(20-4x)$ (M1) for valid approach

$V=8x^2(20-4x)$

$V=160x^2-32x^3$ A1

[2]

(c) By considering the graph of $V=160x^2-32x^3$, the coordinates of the maximum point are $(3.3333342, 592.59259)$. (M1) for valid approach

Thus, the maximum volume is 593 cm^3. A1

[2]

7. (a) By TVM Solver:

$$\boxed{\begin{array}{l} N = 120 \\ I\% = 3.3 \\ PV = 950000 \\ PMT = ? \\ FV = 0 \\ P/Y = 12 \\ C/Y = 12 \\ PMT : END \end{array}}$$

(M1)(A1) for correct values

$PMT = -9305.412721$

Thus, the amount of monthly payment is $\$9310$. A1

[3]

(b) The total amount to be paid

$= (9305.412721)(120)$ (M1) for valid approach

$= \$1116649.527$

$= \$1120000$ A1

[2]

(c) The amount of interest paid

$= 1116649.527 - 950000$ (M1) for valid approach

$= \$166649.5265$

$= \$167000$ A1

[2]

8. (a) 150 A1

[1]

(b) 15 A1

[1]

(c) $y = a(x-(-5))(x-15)$ (A1) for correct approach

$y = a(x+5)(x-15)$

$150 = a(0+5)(0-15)$

$150 = -75a$

$a = -2$ A1

$\therefore y = -2(x+5)(x-15)$

$y = -2(x^2 - 10x - 75)$ (A1) for correct approach

$y = -2x^2 + 20x + 150$

$\therefore b = 20$ A1

[4]

9.	(a)	(i)	420 g	A1	
		(ii)	243 g	A1	
					[2]
	(b)	(i)	1820 g	A1	
		(ii)	40.2 g	A1	
					[2]
	(c)		$Y \sim \mathrm{N}(1820, 1615)$		
			$\mathrm{P}(Y \geq 1770)$		
			$= 0.8932835503$	(A1) for correct value	
			$= 0.893$	A1	
					[2]
10.	(a)		$W = k\sqrt[3]{A}$, where $k \neq 0$	(M1) for valid approach	
			$96 = k\sqrt[3]{512}$		
			$k = 12$		
			$\therefore W = 12\sqrt[3]{A}$	A1	
					[2]
	(b)		$125\ \mathrm{cm}^2$	A1	
					[1]
	(c)		Vertical stretch of scale factor 2	A1	
			followed by translate upward by 7 units.	A1	
					[2]

11. (a) $X \sim \text{Po}(\lambda)$

$P(X = 25) = 0.0555460$

$P(X = 25) - 0.0555460 = 0$ (A1) for correct approach

By considering the graph of

$y = P(X = 25) - 0.0555460$, $\lambda = 21.000003$.

$\therefore \lambda = 21$ A1

[2]

(b) (i) $P(X \geq 19)$

$= 1 - P(X \leq 18)$ (M1) for valid approach

$= 1 - 0.301680304$

$= 0.698319696$

$= 0.698$ A1

(ii) $Y \sim \text{Po}\left(\frac{21}{7}\right)$ (M1) for valid approach

$P(X = 1)$

$= 0.1493612051$

$= 0.149$ A1

(iii) The required probability

$= 0.1493612051^4$ (M1) for valid approach

$= 0.0004976812006$

$= 0.000498$ A1

[6]

12. (a) By considering the graph of $y = 8e^t \sin 3t$, the maximum distance — (M1) for valid approach

$= 115.8163$ cm

$= 116$ cm — A1

[2]

(b) (i) By considering the graph of $y = 8e^t \sin 3t$, the particle first goes back to O at 1.0471976 s. — (M1) for valid approach

Thus, the required time is 1.05 s. — A1

(ii) $s'(t)$

$= (8e^t)(\sin 3t) + (8e^t)(3\cos 3t)$ — (M1) for product rule

$= 8e^t(\sin 3t + 3\cos 3t)$ — A1

(iii) $s''(1.0471976)$

$= -136.783 \text{ cms}^{-2}$

$= -137 \text{ cms}^{-2}$ — A1

[5]

13. (a) (i) $H_0: \ \mu_d = 0$ — A1

(ii) $H_1: \ \mu_d < 0$ — A1

[2]

(b) The p-value

$= 0.1427954705$ — (A1) for correct value

$= 0.143$ — A1

[2]

(c) The null hypothesis is not rejected. — A1

As $p\text{-value} > 0.05$. — R1

[2]

14. (a) $h(x) = g(f(x))$ — (M1) for composite function

$h(x) = 2\sin\left(\frac{f(x)}{3}\right) - 6$ — (A1) for substitution

$h(x) = 2\sin\left(\frac{9x+1}{3}\right) - 6$

$h(x) = 2\sin\left(3x + \frac{1}{3}\right) - 6$ — A1

[3]

(b) The period of h

$= 2\pi \div 3$ — (M1) for valid approach

$= \frac{2\pi}{3}$ — A1

[2]

(c) $\{y : -8 \le y \le -4\}$ — A2

[2]

15. (a) (i) 1 — A1

(ii) $\frac{5}{16}$ — A1

[2]

(b) $f(x) = a\left(x - \left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)\right)\left(x - \left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)\right)$ — (M1) for valid approach

$f(x) = a\left(x^2 - \left(\left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right) + \left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)\right)x + \left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)\left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)\right)$ — (A1) for correct approach

$f(x) = a\left(x^2 - x + \frac{5}{16}\right)$ — A1

[3]

(c) $\frac{5}{2} = a\left(1^2 - 1 + \frac{5}{16}\right)$ — (M1) for setting equation

$\frac{5}{2} = \frac{5}{16}a$

$a = 8$ — A1

[2]

16. (a) The required value

$=V(11)$

$=\frac{1000000}{1+29e^{-2.175}}(11+15)$ (M1) for substitution

$=\$6054063.077$

$=\$6050000$ A1

[2]

(b) $V(t)=10000000$

$\frac{30000000}{1+29e^{-0.145t}}=10000000$ (M1) for setting equation

$\frac{30000000}{1+29e^{-0.145t}}-10000000=0$

By considering the graph of

$y=\frac{30000000}{1+29e^{-0.145t}}-10000000$, $t=18.442404$.

$\therefore t=18.4$ A1

[2]

(c) The value of the pendulum clock will approach $30000000 after a long period of time. R1

[1]

17. (a) (i) $y=e^{0.25x}-1.25$

$y+1.25=e^{0.25x}$ M1

$\ln(y+1.25)=0.25x$ A1

$x=4\ln(y+1.25)$ AG

(ii) The area of R

$=\int_0^8 |4\ln(y+1.25)|\,dy$ M1A1

$=49.19535365$

$=49.2$ A1

[5]

(b) The volume of the solid model

$=\int_0^8 \pi(4\ln(y+1.25))^2\,dy$ (A1) for correct approach

$=1061.499867$

$=1060$ A1

[2]

18. (a) A confidence interval with a smaller confidence level has a narrower interval about the mean. R1

[1]

(b) $(31.1, 44.9)$ A1

[1]

(c) $13.8 = 2(2.575829303)\left(\frac{\sigma}{\sqrt{11}}\right)$ M1A1

$\sigma = 8.884405122$ (A1) for correct value

$\therefore \sigma^2 = 78.93265438$

$\sigma^2 = 78.9$ A1

[4]

AI HL Practice Set 1 Paper 2 Solution

1. (a) $3x+y-10$

$=3(3)+1-10$ A1

$=0$

Thus, P lies on L_1. AG

[1]

(b) 10 A1

[1]

(c) (i) The coordinates of M

$=\left(\frac{3+11}{2},\frac{1+(-3)}{2}\right)$ (A1) for substitution

$=(7,-1)$ A1

(ii) The gradient of PQ

$=\frac{-3-1}{11-3}$ (A1) for substitution

$=-\frac{1}{2}$ A1

(iii) The distance between P and Q

$=\sqrt{(11-3)^2+(-3-1)^2}$ (A1) for substitution

$=8.94427191$

$=8.94$ A1

[6]

(d) The gradient of L_1

$=-\frac{3}{1}$

$=-3$ A1

$\because -3\times-\frac{1}{2}$ M1

$=\frac{3}{2}$

$\neq -1$

Thus, L_1 and L_2 are not perpendicular. AG

[2]

(e) The gradient of L_3

$= \frac{-1}{-3}$ M1

$= \frac{1}{3}$ A1

The equation of L_3:

$y - 1 = \frac{1}{3}(x - 3)$ A1

$3y - 3 = x - 3$ A1

$x - 3y = 0$ AG

[4]

(f) The coordinates of S are $(0, 0)$. (A1) for correct value

The area of the triangle PRS

$= \frac{(10-0)(3-0)}{2}$ (M1) for valid approach

$= 15$ A1

[3]

2.	(a)	(i)	$a = 14.02298851$	
			$a = 14.0$	A1
			$b = -420.2413793$	
			$b = -420$	A1
		(ii)	The estimated pulse rate	
			$= 14.02298851(37) - 420.2413793$	(A1) for substitution
			$= 98.60919557$ beats per minute	
			$= 98.6$ beats per minute	A1
				[4]
	(b)	(i)	$r = 0.592701087$	
			$r = 0.593$	A1
		(ii)	Moderate, Positive	A2
				[3]
	(c)	(i)	H_0: The number of students in each range of pulse rates are evenly distributed.	A1
		(ii)	$p\text{-value} = 0.0166229271$	(A1) for correct value
			$p\text{-value} = 0.0166$	A1
		(iii)	The null hypothesis is rejected.	A1
			As $p\text{-value} < 0.05$.	R1
				[5]
	(d)	(i)	$H_1: \mu_A \neq \mu_B$	A1
		(ii)	$p\text{-value} = 0.3065878383$	(A1) for correct value
			$p\text{-value} = 0.307$	A1
		(iii)	The null hypothesis is not rejected.	A1
			As $p\text{-value} > 0.01$.	R1
				[5]

3. (a) 2 A1

[1]

(b) $f(3)=\frac{4}{3}(3)^3+5(3)^2-6(3)+2$ (M1) for substitution

$f(3)=65$ A1

[2]

(c) $f'(x)=\frac{4}{3}(3x^2)+5(2x)-6(1)+0$ (A1) for correct derivatives

$f'(x)=4x^2+10x-6$ A1

[2]

(d) $4x^2+10x-6=0$

$2(x+3)(2x-1)=0$ (M1) for valid approach

$x=-3$ or $x=\frac{1}{2}$ A2

[3]

(e) $y=29$, $y=\frac{5}{12}$ A2

[2]

(f) (i) $\frac{5}{12}<w<29$ A2

(ii) $w<\frac{5}{12}$ or $w>29$ A2

[4]

(g) The gradient of the tangent

$=f'(3)$

$=4(3)^2+10(3)-6$ (A1) for substitution

$=60$ A1

[2]

(h) The equation of the normal:

$y-65=\frac{-1}{60}(x-3)$ M1A1

$-60y+3900=x-3$ A1

$x+60y-3903=0$ AG

[3]

4. (a) (i) 4 A1

(ii) 2 A1

(iii) 4 A1

[3]

(b) AB A1

[1]

(c) For any three edges correct A1
For all edges correct A1

1. Choose AB of weight 10
2. Choose BC of weight 15
3. Choose AF of weight 18
4. Choose BE of weight 18
5. Choose CD of weight 20

Thus, the minimum spanning tree is a tree containing AB, BC, AF, BE and CD. A1

[3]

(d) 81 A1

[1]

(e) For any four edges correct A1
For any eight edges correct A1

1. Choose CD of weight 20
2. Choose DE of weight 25
3. Choose EF of weight 23
4. Choose FA of weight 18
5. Choose AB of weight 10
6. Choose BC of weight 15
7. Choose CE of weight 30
8. Choose EB of weight 18
9. Choose BF of weight 27
10. Choose FB of weight 27
11. Choose BC of weight 15

Thus, a possible route contains CD, DE, EF, FA, AB, BC, CE, EB, BF, FB and BC. A1

[3]

(f) 228 A1

[1]

5. (a) (i) $\mathbf{M}^2$

$=\begin{pmatrix}1 & 0.5\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0.5\\0 & 1\end{pmatrix}$

$=\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$ A2

(ii) $\mathbf{M}^3$

$=\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0.5\\0 & 1\end{pmatrix}$

$=\begin{pmatrix}1 & 1.5\\0 & 1\end{pmatrix}$ A2

(iii) $\mathbf{M}^{30}=\begin{pmatrix}1 & 15\\0 & 1\end{pmatrix}$ A1

[5]

(b) (i) $s(2)=\begin{pmatrix}2 & 1.5\\0 & 2\end{pmatrix}$ A1

(ii) $s(3)=\begin{pmatrix}3 & 3\\0 & 3\end{pmatrix}$ A1

(iii) $s(30)$

$=\begin{pmatrix}1 & 0.5\\0 & 1\end{pmatrix}+\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}+\cdots+\begin{pmatrix}1 & 15\\0 & 1\end{pmatrix}$

$=\begin{pmatrix}30 & 0.5+1+\cdots+15\\0 & 30\end{pmatrix}$ (M1) for valid approach

$=\begin{pmatrix}30 & \frac{30}{2}(0.5+15)\\0 & 30\end{pmatrix}$ M1A1

$=\begin{pmatrix}30 & 232.5\\0 & 30\end{pmatrix}$ A1

[6]

(c) $r(10)$

$$=\begin{pmatrix}1 & 0.5\\0 & 1\end{pmatrix}+\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}+\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}+\begin{pmatrix}1 & 4\\0 & 1\end{pmatrix}$$

$$+\cdots+\begin{pmatrix}1 & 0.5\cdot 2^{10-1}\\0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}10 & 0.5+1+\cdots+0.5\cdot 2^{9}\\0 & 10\end{pmatrix}$$ (M1) for valid approach

$$=\begin{pmatrix}10 & \frac{0.5(1-2^{10})}{1-2}\\0 & 10\end{pmatrix}$$ M1A1

$$=\begin{pmatrix}10 & \frac{1023}{2}\\0 & 10\end{pmatrix}$$ A1

[4]

6. (a) $\begin{cases} \dfrac{dv}{dt} = 25x \\ \dfrac{dx}{dt} = v \end{cases}$ A1

[1]

(b) (i) $\begin{cases} v_{n+1} = v_n + 0.2 \left.\dfrac{dv}{dt}\right|_{(t_n, v_n, x_n)} \\ x_{n+1} = x_n + 0.2 \left.\dfrac{dx}{dt}\right|_{(t_n, v_n, x_n)} \\ t_{n+1} = t_n + 0.2 \end{cases}$ (M1) for valid approach

$t_0 = 0$, $v_0 = 0$, $x_0 = 1$ (A1) for correct values

$t_1 = 0 + 0.2 = 0.2$

$v_1 = 0 + 0.2(25) = 5$ A1

(ii) $x_1 = 1 + 0.2(0) = 1$ A1

[4]

(c) (i) 2 cm A1

(ii) 16 cm A1

(iii) 4096 cm A1

[3]

(d) $\det(\mathbf{M} - \lambda\mathbf{I})$

$= \begin{vmatrix} 0-\lambda & 25 \\ 1 & 0-\lambda \end{vmatrix}$ (M1) for valid approach

$= (-\lambda)(-\lambda) - (25)(1)$

$= \lambda^2 - 25$ A1

[2]

(e) $\lambda_1 = -5$, $\lambda_2 = 5$ A2

[2]

(f) $\mathbf{v}_1 = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ A2

[2]

(g) $\mathbf{X} = Ae^{-5t}\begin{pmatrix}-5\\1\end{pmatrix} + Be^{5t}\begin{pmatrix}5\\1\end{pmatrix}$ A1

$\begin{pmatrix}0\\1\end{pmatrix} = Ae^{-5(0)}\begin{pmatrix}-5\\1\end{pmatrix} + Be^{5(0)}\begin{pmatrix}5\\1\end{pmatrix}$ M1

$\begin{cases}0 = -5A + 5B\\ 1 = A + B\end{cases}$

By solving this system, $A = 0.5$ and $B = 0.5$. A1

Thus, the particular solution of x is given by

$x = 0.5e^{-5t} + 0.5e^{5t}$. AG

[3]

7. (a) $\mathbf{r}=\begin{pmatrix}5\\5\\0\end{pmatrix}+t\begin{pmatrix}-10\\10\\0\end{pmatrix}$ A2

[2]

(b) $-85=5-10p$ (M1) for setting equation

$-90=-10p$

$p=9$ A1

[2]

(c) The velocity vector of B

$=\frac{1}{5}\left(\begin{pmatrix}-50\\50\\0\end{pmatrix}-\begin{pmatrix}0\\0\\-50\end{pmatrix}\right)$ (M1) for valid approach

$=\begin{pmatrix}-10\\10\\10\end{pmatrix}\mathrm{s}^{-1}$ A1

[2]

(d) $\mathbf{r}=\begin{pmatrix}0\\0\\-50\end{pmatrix}+t\begin{pmatrix}-10\\10\\10\end{pmatrix}$ A2

[2]

(e) $\mathbf{r}_\mathrm{A}=\begin{pmatrix}5-10t\\5+10t\\0\end{pmatrix}$, $\mathbf{r}_B=\begin{pmatrix}-10t\\10t\\-50+10t\end{pmatrix}$ (A1) for correct values

$\mathbf{r}_\mathrm{A}-\mathbf{r}_B$

$=\begin{pmatrix}5-10t\\5+10t\\0\end{pmatrix}-\begin{pmatrix}-10t\\10t\\-50+10t\end{pmatrix}$

$=\begin{pmatrix}5\\5\\50-10t\end{pmatrix}$ (A1) for correct value

$|\mathbf{r}_\mathrm{A}-\mathbf{r}_B|=\sqrt{5^2+5^2+(50-10t)^2}$ (A1) for correct approach

By considering the graph of $y=\sqrt{50+(50-10t)^2}$,

the minimum point is $(5.0000005, 7.0710678)$.

Thus, the shortest distance is 7.07. A1

[4]

(f) 5.00 seconds after the start of the game A1

[1]

AI HL Practice Set 1 Paper 3 Solution

1. (a) (i) $\tan\frac{\pi}{6}=\frac{\text{DE}}{30}$ (M1) for tangent ratio

$\text{DE}=17.32050808\text{ m}$

$\text{DE}=17.3\text{ m}$ A1

(ii) The area of the triangle ODE

$=\frac{(30)(17.32050808)}{2}$ A1

$=259.8076212\text{ m}^2$

$=260\text{ m}^2$ AG

(iii) 1.46 A1

[4]

(b) (i) $\frac{(30)(\text{DE})}{2}=\frac{(30)(30)}{3}$ (M1) for setting equation

$\text{DE}=20\text{ m}$ A1

(ii) $\tan\text{D}\hat{\text{O}}\text{E}=\frac{20}{30}$ (M1) for tangent ratio

$\text{D}\hat{\text{O}}\text{E}=0.5880026035\text{ rad}$

$\text{D}\hat{\text{O}}\text{E}=0.588\text{ rad}$ A1

(iii) 0.395 rad A1

[5]

(c) (i) BD and CF are perpendicular. A1

(ii) The required coordinates

$=\left(\frac{20+30}{2},\frac{30+20}{2}\right)$ (A1) for substitution

$=(25,25)$ A1

(iii) $(20,20)$ A2

[5]

(d) $\mathbf{M}=\begin{pmatrix}0&1&1&0&1&1\\1&0&1&0&0&0\\1&1&0&1&1&0\\0&0&1&0&1&0\\1&0&1&1&0&1\\1&0&0&0&1&0\end{pmatrix}$ A3

[3]

(e) $\mathbf{M}+\mathbf{M}^2+\mathbf{M}^3=\begin{pmatrix}10&8&12&6&12&8\\8&4&8&4&6&4\\12&8&10&8&12&6\\6&4&8&4&8&4\\12&6&12&8&10&8\\8&4&6&4&8&4\end{pmatrix}$ (M1) for valid approach

Thus, the total number of walks of length at most 3 from C to E is 4. A1

[2]

(f) (i) 46.1 A1

(ii) 54.1 A1

[2]

(g) For any three edges correct A1

For all edges correct A1

1. Choose OA of distance 30
2. Choose AB of distance 20
3. Choose BC of distance 10
4. Choose CD of distance 10
5. Choose DE of distance 20
6. Choose EO of distance 30

Thus, the required upper bound is $120\ \mathrm{m}$. A1

[3]

(h) For any two edges correct A1

For all edges correct A1

1. Choose BD of distance 14.1
2. Choose AB of distance 20
3. Choose DE of distance 20
4. Choose OA of distance 30

Therefore, the distance of a minimum spanning tree after deleting the vertex C is 84.1. A1

The required lower bound

$=84.1+10+10$

$=104.1\,\mathrm{m}$ A1

[4]

2. (a) (i) The required probability

$$=\left(\frac{45+35+20}{300}\right)\left(\frac{45+35+20-1}{300-1}\right)$$ (M1) for valid approach

$$=\frac{33}{299}$$ A1

(ii) The required probability

$$=\frac{\left(\frac{45}{300}\right)\left(\frac{45-1}{300-1}\right)+\left(\frac{35}{300}\right)\left(\frac{35-1}{300-1}\right)+\left(\frac{20}{300}\right)\left(\frac{20-1}{300-1}\right)}{\frac{33}{299}}$$ M1A1

$$=\frac{71}{198}$$ A1

[5]

(b) (i) $H_0:\ p=0.18$ A1

(ii) $H_1:\ p>0.18$ A1

(iii) $\mathrm{P}(X\geq 7)$

$=1-\mathrm{P}(X\leq 6)$ (M1) for valid approach

$=0.148763448$

Thus, the p-value is 0.149. A1

(iv) The null hypothesis is not rejected. A1

As $p\text{-value}>0.05$. R1

[6]

(c) (i) 48.6 A1

(ii) 19.6 A1

(iii) 385 A1

[3]

(d) (i) H_0: The data follows a normal distribution with parameters $\mathrm{N}(48.6, 19.6126367^2)$. A1

(ii) 16.4 A1

(iii) 2 A1

(iv) $p\text{-value} = 0.0004378451724$ (A1) for correct value

$p\text{-value} = 0.000438$ A1

(v) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1

[7]

(e) (i) $H_0:\ \lambda = 11$ A1

(ii) $H_1:\ \lambda < 11$ A1

[2]

(f) The required probability

$= \mathrm{P}(X \le 5 \mid \lambda = 11)$ (M1) for valid approach

$= 0.0375198141$

$= 0.0375$ A1

[2]

(g) The required probability

$= \mathrm{P}(X \ge 6 \mid \lambda = 7)$ (M1) for valid approach

$= 1 - \mathrm{P}(X \le 5 \mid \lambda = 7)$

$= 0.6992917238$

$= 0.699$ A1

[2]

AI HL Practice Set 2 Paper 1 Solution

1. (a) (i) 40 A1

(ii) 1 A1

(iii) 0 A1

[3]

(b) The mean number of watermelons

$$=\frac{(0)(12)+(1)(10)+(2)(6)+(3)(5)+(4)(5)+(5)(2)}{12+10+6+5+5+2}$$ (A1) for correct formula

$=1.675$ A1

[2]

(c) Discrete A1

[1]

2. (a) (i) 3.5 A1

(ii) 9.5 A1

(iii) 2.5 A1

[3]

(b) The period of d

$$=\frac{360^\circ}{3^\circ}$$ (M1) for valid approach

$=120\text{ minutes}$ A1

[2]

(c) $10:30\text{ am}$ A1

[1]

3. (a) (i) x_n A1

(ii) z_n A1

[2]

(b) The required term

$= 100 + (10 - 1)(200)$ (A1) for substitution

$= 1900$ A1

[2]

(c) The required sum

$= \dfrac{100(3^{10} - 1)}{3 - 1}$ (A1) for substitution

$= 2952400$ A1

[2]

4. (a) (i) The required radius

$= \sqrt{(12-8)^2 + (14-11)^2}$ (A1) for substitution

$= 5$ A1

(ii) The required radius

$= \sqrt{\left(6 - \frac{41}{7}\right)^2 + \left(2 - \frac{57}{7}\right)^2}$ (A1) for substitution

$= 6.144518048$

$= 6.14$ A1

[4]

(b) F A1

[1]

5. (a) By TVM Solver:

$N = ?$
$I\% = 2.95$
$PV = 120000$
$PMT = -2000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
$PMT: END$

(M1)(A1) for correct values

$N = 64.99449865$

Thus, the number of months to repay the loan is 65 months. A1

[3]

(b) The amount of interest paid

$= (2000)(65) - 120000$ (M1)(A1) for substitution

$= \$10000$ A1

[3]

6. (a) The required cost

$= \frac{1}{2}(100-90)^2 + 60$ (M1) for substitution

$= \$110$ A1

[2]

(b) $C(x) \le 1310$

$\frac{1}{2}(x-90)^2 + 60 \le 1310$ (M1) for setting inequality

$\frac{1}{2}(x-90)^2 - 1250 \le 0$

By considering the graph of

$y = \frac{1}{2}(x-90)^2 - 1250$, $40 \le x \le 140$.

$\therefore n = 40$ A1

[2]

(c) The minimum point of the graph of $C(x)$ is

$(90, 60)$. (M1) for valid approach

Thus, the required number of jackets is 90. A1

[2]

7. (a) (i) 0.683 A1

(ii) 0.954 A1

[2]

(b) $\mathrm{P}(H < 2.82)$

$= 0.4372698598$ (A1) for correct value

$= 0.437$ A1

[2]

(c) $\mathrm{P}(H > r) = 0.28$ (M1) for valid approach

$\mathrm{P}(H < r) = 0.72$

$r = 2.960739885$

$r = 2.96$ A1

[2]

8. (a) $\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 - 2(\mathrm{AB})(\mathrm{BC})\cos \mathrm{A\hat{B}C}$ (M1) for cosine rule

$\mathrm{AC}^2 = 15^2 + 13.5^2 - 2(15)(13.5)\cos 98^\circ$ (A1) for substitution

$\mathrm{AC} = 21.53172324\ \mathrm{m}$

$\mathrm{AC} = 21.5\ \mathrm{m}$ A1

[3]

(b) $\dfrac{\sin \mathrm{B\hat{A}C}}{\mathrm{BC}} = \dfrac{\sin \mathrm{A\hat{B}C}}{\mathrm{AC}}$ (M1) for sine rule

$\dfrac{\sin \mathrm{B\hat{A}C}}{13.5} = \dfrac{\sin 98^\circ}{21.53172324}$ (A1) for substitution

$\sin \mathrm{B\hat{A}C} = \dfrac{13.5 \sin 98^\circ}{21.53172324}$

$\mathrm{B\hat{A}C} = 38.38043409^\circ$

$\mathrm{B\hat{A}C} = 38.4^\circ$ A1

[3]

9. (a) $X \sim \text{Po}(3.3)$

$\text{P}(X < 3)$

$= \text{P}(X \le 2)$ (M1) for valid approach

$= 0.3594264663$

$= 0.359$ A1

[2]

(b) $Y \sim \text{Po}(9.9)$ (M1) for valid approach

$\text{P}(Y = 10)$

$= 0.1250470764$

$= 0.125$ A1

[2]

(c) $\text{P}(Y < 14 \mid Y > 9)$

$= \dfrac{\text{P}(Y < 14 \cap Y > 9)}{\text{P}(Y > 9)}$ (A1) for substitution

$= \dfrac{\text{P}(10 \le Y \le 13)}{1 - \text{P}(Y \le 9)}$

$= \dfrac{0.4011438055}{0.5294984163}$ (A1) for correct approach

$= 0.757592078$

$= 0.758$ A1

[3]

10. (a) $W = hk^x$

$\ln W = \ln(hk^x)$ (A1) for correct approach

$\ln W = \ln h + \ln k^x$ (A1) for correct approach

$\ln W = (\ln k)x + \ln h$ A1

[3]

(b) (i) $\ln h = -0.85$

$h = e^{-0.85}$ (M1) for valid approach

$h = 0.4274149319$

$h = 0.42741$ A1

(ii) $\ln k = 0.4$

$k = e^{0.4}$ (M1) for valid approach

$k = 1.491824698$

$k = 1.4918$ A1

[4]

11. (a) $3\mathbf{a}+2\mathbf{b}=\begin{pmatrix}2\\18\\19\end{pmatrix}$ A1

[1]

(b) (i) The required component

$=\frac{(3\mathbf{a}+2\mathbf{b})\cdot(\mathbf{a})}{|\mathbf{a}|}$ (M1) for valid approach

$=\frac{(2)(2)+(18)(4)+(19)(3)}{\sqrt{2^2+4^2+3^2}}$ (A1) for substitution

$=24.69747998$

$=24.7$ A1

(ii) The required component

$=\frac{|(3\mathbf{a}+2\mathbf{b})\times\mathbf{b}|}{|\mathbf{b}|}$ (M1) for valid approach

$=\frac{\left|\begin{pmatrix}(18)(5)-(19)(3)\\(19)(-2)-(2)(5)\\(2)(3)-(18)(-2)\end{pmatrix}\right|}{\sqrt{(-2)^2+3^2+5^2}}$ (A1) for substitution

$=\frac{\sqrt{33^2+(-48)^2+42^2}}{\sqrt{(-2)^2+3^2+5^2}}$

$=11.6494861$

$=11.6$ A1

[3]

12. (a) $\mathrm{E}(X)$

$=(3)(0.3)+(5)(0.1)+(7)(0.15)+(9)(0.45)$ (A1) for substitution

$=6.5$ A1

[2]

(b) $\mathrm{E}(2X-5Y)$

$=2(6.5)-5(17)$ (A1) for substitution

$=-72$ A1

[2]

(c) $\mathrm{Var}(2X-5Y)$

$=2^2\,\mathrm{Var}(X)+5^2\,\mathrm{Var}(Y)$

$=4(6.75)+25(3)$ (A1) for substitution

$=102$ A1

[2]

13. (a) 700 A1

[1]

(b) $\text{Var}(\bar{X})$

$= \dfrac{\text{Var}(X)}{n}$

$= \dfrac{15.5}{320}$ (A1) for substitution

$= \dfrac{31}{640}$ A1

[2]

(c) $\bar{X} \sim \text{N}\left(700, \dfrac{31}{640}\right)$ (M1) for valid approach

$\text{P}(\bar{X} < 699.83)$

$= 0.2199303896$

$= 0.220$ A1

[2]

14. (a) The required number of leopards

$= w(2)$ (A1) for correct approach

$= 237\cos 0.5(2) + 850$ (A1) for substitution

$= 978.0516465$

$= 978$ A1

[3]

(b) $\dfrac{\text{d}w}{\text{d}t}$

$= 237(-\sin 0.5t)(0.5) + 0$ (M1) for chain rule

$= -118.5\sin 0.5t$ A1

[2]

(c) By considering the graph of

$y = -118.5\sin 0.5t$, $\dfrac{\text{d}w}{\text{d}t}$ attains its maximum

for the first time when $t = 9.4247780$. (A1) for correct value

The value of n

$= (9.4247780)(30)$ (A1) for correct approach

$= 282.74334$

$= 283$ A1

[3]

15. (a) $(2, 0)$ A1

[1]

(b) 2 A1

[1]

(c) $y = ((x+4)^2 - 36)^2$

$\Rightarrow x = ((y+4)^2 - 36)^2$ (M1) for swapping variables

$\sqrt{x} = (y+4)^2 - 36$

$(y+4)^2 = \sqrt{x} + 36$

$y + 4 = \sqrt{\sqrt{x} + 36}$

$y = \sqrt{\sqrt{x} + 36} - 4$ (M1) for valid approach

$\therefore f^{-1}(x) = \sqrt{\sqrt{x} + 36} - 4$ A1

[3]

16. (a) (i) $z_1^{\,5}$

$= \left(\frac{1}{2}\operatorname{cis}\frac{\pi}{10}\right)^5$

$= \left(\frac{1}{2}\right)^5 \operatorname{cis}\left(5\left(\frac{\pi}{10}\right)\right)$ (M1) for valid approach

$= \frac{1}{32}\operatorname{cis}\frac{\pi}{2}$ A1

(ii) 0 A1

[3]

(b) (i) $\frac{z_1^{\,5}}{z_2}$

$= \left(\frac{1}{32}\operatorname{cis}\frac{\pi}{2}\right) \div \left(\frac{1}{8}\operatorname{cis}\frac{\pi}{4}\right)$

$= \left(\frac{1}{32} \div \frac{1}{8}\right)\operatorname{cis}\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$ (M1) for valid approach

$= \frac{1}{4}\operatorname{cis}\frac{\pi}{4}$ A1

(ii) $\frac{1}{4}e^{\frac{\pi}{4}\mathrm{i}}$ A1

[3]

17. (a) (i) $y = 2.02 \cdot 1.45^x$ A2

(ii) $R^2 = 0.8543621308$

$R^2 = 0.85436$ A1

[3]

(b) $SS_{res} = 7.102577562$

$SS_{res} = 7.10$ A2

[2]

(c) $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$

$0.8543621308 = 1 - \frac{7.102577562}{SS_{tot}}$ (A1) for substitution

$\frac{7.102577562}{SS_{tot}} = 0.1456378692$

$SS_{tot} = 48.768755$

$SS_{tot} = 48.8$ A1

[2]

18. (a) $x > 4$ A1

[1]

(b) $$\begin{cases} x_{n+1} = x_n + 0.05 \left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n,\, x_n,\, y_n)} \\ y_{n+1} = y_n + 0.05 \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{(t_n,\, x_n,\, y_n)} \\ t_{n+1} = t_n + 0.05 \end{cases}$$ (M1) for valid approach

$t_0 = 0$, $x_0 = 4.5$, $y_0 = 4.5$ (A1) for correct values

$t_1 = 0 + 0.05 = 0.05$

$y_1 = 4.5 + 0.05\big((4(4.5) - 16)(4.5)\big) = 4.95$ (A1) for correct value

Thus, the approximate numbers of soldiers from country Y is 4950. A1

[4]

AI HL Practice Set 2 Paper 2 Solution

1. (a) $7(98)+24f-2990=0$ (M1) for setting equation

$24f=2304$

$f=96$ A1

[2]

(b) $-\frac{7}{24}$ A1

[1]

(c) (i) The gradient of DE

$=-1\div-\frac{7}{24}$ (M1) for valid approach

$=\frac{24}{7}$ A1

(ii) The equation of DE:

$y-10=\frac{24}{7}(x-125)$ M1A1

$7y-70=24(x-125)$ A1

$7y-70=24x-3000$

$24x-7y-2930=0$ AG

[5]

(d) $(146, 82)$ A2

[2]

(e) The coordinates of the mid-point of CD

$=\left(\frac{50+146}{2},\frac{110+82}{2}\right)$ M1A1

$=(98, 96)$

Thus, F is the mid-point of CD. AG

[2]

(f) The length of DE

$=\sqrt{(146-125)^2+(82-10)^2}$ (A1) for substitution

$=75$ A1

[2]

(g) The area of the triangle CDE

$= \dfrac{(75)(100)}{2}$ (M1) for valid approach

$= 3750\ \text{m}^2$ A1

[2]

(h) The total area

$= 3750 + \dfrac{(\text{BC}+\text{AE})(\text{AB})}{2}$ (M1)(A1) for correct approach

$= 3750 + \dfrac{(40+115)(100)}{2}$ (A1) for substitution

$= 11500\ \text{m}^2$ A1

[4]

2. (a) $H_1:\ \mu_1 > \mu_2$ A1

[1]

(b) $p\text{-value} = 0.0231895114$ (A1) for correct value

$p\text{-value} = 0.0232$ A1

[2]

(c) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1

[2]

(d) (i) The required probability

$= \left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$= \frac{1}{9}$ A1

(ii) The required probability

$= \left(\frac{5}{10}\right)\left(\frac{2}{9}\right) + \left(\frac{5}{10}\right)\left(\frac{7}{9}\right) + \left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$= \frac{11}{18}$ A1

[4]

(e) H_1: The age and the reading preference are not independent. A1

[1]

(f) 4 A1

[1]

(g) $\chi^2_{calc} = 53.64204545$ (A1) for correct value

$\chi^2_{calc} = 53.6$ A1

[2]

(h) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 13.277$. R1

[2]

3.	(a)	$f'(x) = -3x^2 + b(2x) - 432(1) + 0$	(A1) for correct derivatives
		$f'(x) = -3x^2 + 2bx - 432$	
		$f'(8) = 0$	(M1) for setting equation
		$\therefore -3(8)^2 + 2b(8) - 432 = 0$	(A1) for substitution
		$16b = 624$	
		$b = 39$	A1
			[4]
	(b)	(i) 984	A1
		(ii) $(18, 1484)$	A2
			[3]
	(c)	$8 < x < 18$	A2
			[2]
	(d)	(i) $984 < k < 1484$	A2
		(ii) $k \le 984$ or $k \ge 1484$	A2
			[4]
	(e)	$C(x) = -x^3 + 39x^2 - 432x + 2456$	
		$C(8) = 984$	
		$C(25)$	
		$= -25^3 + 39(25)^2 - 432(25) + 2456$	A1
		$= 406$	
		$C(8) > C(25)$	R1
		Thus, the average cost attains its minimum when 25000 smart watches are produced.	AG
			[2]
	(f)	$C(x) \le 984$	(M1) for setting inequality
		$-x^3 + 39x^2 - 432x + 2456 \le 984$	
		$-x^3 + 39x^2 - 432x + 1472 \le 0$	
		By considering the graph of $y = -x^3 + 39x^2 - 432x + 1472$, $x = 8$ or $x \ge 23$.	
		Thus, the range of values of x are $x = 8$ or $23 \le x \le 25$.	A2
			[3]

4. (a) The initial velocity

$= v(0)$

$= -0.5(0-5)^3$ (M1) for substitution

$= 62.5 \text{ ms}^{-1}$ A1

[2]

(b) $v(t) = -13.5$ (M1) for setting equation

$-0.5(t-5)^3 = -13.5$

$(t-5)^3 = 27$

$t - 5 = 3$ (A1) for correct approach

$t = 8$ A1

[3]

(c) The total distance travelled

$= \int_0^{10} |v(t)| \, \mathrm{d}t$ (M1) for valid approach

$= \int_0^{10} \left| -0.5(t-5)^3 \right| \mathrm{d}t$ (A1) for substitution

$= 156.25 \text{ m}$ A1

[3]

(d) $a(t) = v'(t)$

$a(t) = -0.5(3)(t-5)^2(1)$ (A1) for correct approach

$a(t) = -1.5(t-5)^2$ A1

[2]

(e) $v(t) \geq 0$ and $a(t) \geq 0$

By considering the graph of $y = -0.5(t-5)^3$ and

$y = -1.5(t-5)^2$, $0 \leq t \leq 5$ and $t = 5$. R2

$\therefore t = 5$ A1

[3]

(f) $s(t) = \int v(t) \mathrm{d}t$

$s(t) = \int -0.5(t-5)^3 \mathrm{d}t$ (A1) for correct approach

$s(t) = -0.125(t-5)^4 + C$ A1

$-78 = -0.125(0-5)^4 + C$ (M1) for substitution

$C = 0.125$

$\therefore s(t) = -0.125(t-5)^4 + 0.125$ A1

[4]

5. (a) $L_1:\begin{cases} x = 3+2t \\ y = 6-6t \\ z = 9-2t \end{cases}$, $L_2:\begin{cases} x = 1+3s \\ y = -2-2s \\ z = 3+s \end{cases}$ M1

$9-2t = 3+s$

$s = 6-2t$

$3+2t = 1+3s$

$\therefore 3+2t = 1+3(6-2t)$ M1

$3+2t = 19-6t$

$8t = 16$

$t = 2$

$\therefore s = 6-2(2) = 2$ A1

$\begin{cases} x = 3+2(2) = 7 \\ y = 6-6(2) = -6 \\ z = 9-2(2) = 5 \end{cases}$ M1

Thus, the coordinates of C are $(7, -6, 5)$. AG

[4]

(b) $(3\mathbf{i}-2\mathbf{j}+\mathbf{k})\cdot\mathbf{k} = |3\mathbf{i}-2\mathbf{j}+\mathbf{k}||\mathbf{k}|\cos\theta$ (M1) for valid approach

$(3)(0)+(-2)(0)+(1)(1) = (\sqrt{3^2+(-2)^2+1^2})(1)\cos\theta$ (A1) for correct approach

$1 = \sqrt{14}\cos\theta$

$\cos\theta = \frac{1}{\sqrt{14}}$

$\theta = 1.300246564 \text{ rad}$

$\theta = 1.30 \text{ rad}$ A1

[3]

(c) (i) $\vec{CA} = 6\mathbf{i} - 18\mathbf{j} - 6\mathbf{k}$ A1

(ii) $\vec{CB} = 9\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ A1

(iii) The required area

$= \frac{1}{2}\left|\vec{CA} \times \vec{CB}\right|$ (M1) for valid approach

$= \frac{1}{2}\left|\begin{pmatrix}(-18)(3)-(-6)(-6)\\(-6)(9)-(6)(3)\\(6)(-6)-(-18)(9)\end{pmatrix}\right|$ (A1) for substitution

$= \frac{1}{2}\left|-90\mathbf{i} - 72\mathbf{j} + 126\mathbf{k}\right|$

$= \frac{1}{2}\sqrt{(-90)^2 + (-72)^2 + 126^2}$

$= 85.38149682$

$= 85.4$ A1

[5]

(d) 171 A1

[1]

6. (a) Eulerian circuit does not exist . A1
As not all vertices are of even degree. R1

[2]

(b) BC A1

[1]

(c) For any three edges correct A1
For all edges correct A1

1. Choose BC of weight 6
2. Choose BG of weight 10
3. Choose GE of weight 11
4. Choose EF of weight 9
5. Choose AB of weight 17
6. Choose ED of weight 19

Thus, the minimum spanning tree is a tree containing BC, BG, GE, EF, AB and ED. A1

[3]

(d) 72 A1

[1]

(e) For any three edges correct A1
For all edges correct A1

1. Choose GB of weight 10
2. Choose BC of weight 6
3. Choose CD of weight 21
4. Choose DE of weight 19
5. Choose EF of weight 9
6. Choose FA of weight 18
7. Choose AG of weight 26

Thus, the required upper bound is 109. A1

[3]

(f) For any two edges correct A1
For all edges correct A1

1. Choose BC of weight 6
2. Choose EF of weight 9
3. Choose AB of weight 17
4. Choose AF of weight 18
5. Choose DE of weight 19

Therefore, the weight of a minimum spanning tree after deleting the vertex G is 69. A1
The required lower bound
$= 69 + 10 + 11$
$= 90$ A1

[4]

7. (a) The characteristic polynomial of $\mathbf{M}$

$=\det(\mathbf{M}-\lambda\mathbf{I})$

$$=\begin{vmatrix}\frac{5}{3}-\lambda & \frac{4}{3}\\ -\frac{2}{3} & -\frac{1}{3}-\lambda\end{vmatrix}$$ (M1) for valid approach

$$=\left(\frac{5}{3}-\lambda\right)\left(-\frac{1}{3}-\lambda\right)-\left(\frac{4}{3}\right)\left(-\frac{2}{3}\right)$$

$$=-\frac{5}{9}-\frac{4}{3}\lambda+\lambda^2+\frac{8}{9}$$

$$=\lambda^2-\frac{4}{3}\lambda+\frac{1}{3}$$ A1

[2]

(b) $\lambda_1=\frac{1}{3}$, $\lambda_2=1$ A2

[2]

(c) $\mathbf{v}_1=\begin{pmatrix}1\\-1\end{pmatrix}$, $\mathbf{v}_2=\begin{pmatrix}1\\-\frac{1}{2}\end{pmatrix}$ A2

[2]

(d) (i) $\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}$ A1

(ii) $\begin{pmatrix}\left(\frac{1}{3}\right)^n & 0\\ 0 & 1\end{pmatrix}$ A2

[3]

(e) $\mathbf{M}^n$

$$=\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}\left(\frac{1}{3}\right)^n & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}^{-1}$$

$$=\begin{pmatrix}\left(\frac{1}{3}\right)^n & 1\\ -\left(\frac{1}{3}\right)^n & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}^{-1}$$ A1

$$=\begin{pmatrix}\left(\frac{1}{3}\right)^n & 1\\ -\left(\frac{1}{3}\right)^n & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}-1 & -2\\ 2 & 2\end{pmatrix}$$ (A1) for correct approach

$$=\begin{pmatrix}-\left(\frac{1}{3}\right)^n+2 & -2\left(\frac{1}{3}\right)^n+2\\ \left(\frac{1}{3}\right)^n-1 & 2\left(\frac{1}{3}\right)^n-1\end{pmatrix}$$ A1

[3]

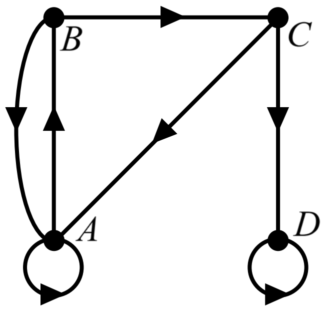
(f) $\lim_{n\to\infty} g(n)=2$ A1

[1]

AI HL Practice Set 2 Paper 3 Solution

1. (a) For correct number of directed edges A1
For correct number of loops A1
For correct directions A2

[4]



(b) The column sum represents the in-degree of the corresponding vertex. A1

[1]

(c) (i) $\mathbf{M}=\begin{pmatrix}1&1&0&0\\1&0&1&0\\1&0&0&1\\0&0&0&1\end{pmatrix}$ A2

(ii) $\mathbf{T}=\begin{pmatrix}\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0\\\frac{1}{2}&0&0&0\\0&\frac{1}{2}&0&0\\0&0&\frac{1}{2}&1\end{pmatrix}$ A2

[4]

(d) (i) The player is definitely at the state A before his tosses the coin for the first time. R1

(ii) $\mathbf{v}_1 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0.5 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix}$ A2

(iii) There are four scenarios that the player will be at the state A after the coin is tossed for three times:
For any two scenarios correct R1
For all scenarios correct R1
1. Getting three consecutive tails
2. Getting one head followed by two consecutive tails
3. Getting heads and tails alternatively, starting with a tail
4. Getting two consecutive heads followed by one tail
Also, as the probability for each scenario is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, $\alpha_1 = 4\left(\frac{1}{8}\right) = \frac{1}{2}$. R1

(iv) $\alpha_2 : \alpha_3 : \alpha_4 = 2:1:1$ A1

[7]

(e) Let $\mathbf{v}=\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}$ be the steady state probability vector, where $e+f+g+h=1$.

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}=\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}$$ M1

$$\begin{pmatrix} \frac{1}{2}e+\frac{1}{2}f+\frac{1}{2}g \\ \frac{1}{2}e \\ \frac{1}{2}f \\ \frac{1}{2}g+h \end{pmatrix}=\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}$$ A1

$\frac{1}{2}g+h=h$

$g=0$

$\frac{1}{2}f=0$

$f=0$

$\frac{1}{2}e=0$

$e=0$ A1

$0+0+0+h=1$ M1

$h=1$

Thus, the steady state probability vector is

$$\mathbf{v}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$ AG

[4]

(f) (i) The required probability

$$=(1)\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^{3}$$ M1

$$=\frac{2}{81}$$ A1

(ii) The required probability

$$=\left(1-\left(\frac{1}{3}\right)^{3}\right)\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^{3}$$ M1A2

$$=\frac{52}{2187}$$ A1

(iii) The required probability

$$=1-\left(\frac{1}{3}\right)^{3}-\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^{3}-\frac{2}{81}$$

M1A2

$$-(1)^{2}\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^{3}-\frac{52}{2187}$$

$$=\frac{1892}{2187}$$ A1

[10]

2.	(a)	(i)	0.212	A1	
		(ii)	$\bar{W} \sim \mathrm{N}\left(300, \frac{10^2}{12}\right)$	A1	
			The required probability		
			$= \mathrm{P}(\bar{W} < 292)$		
			$= 0.002791866$	(A1) for correct value	
			$= 0.00279$	A1	
					[4]
	(b)	(i)	6000 g	A1	
		(ii)	The required variance		
			$= 20(10^2)$	(A1) for substitution	
			$= 2000 \text{ g}^2$	A1	
		(iii)	The required probability		
			$= 0.0126736174$	(A1) for correct value	
			$= 0.0127$	A1	
					[5]
	(c)	(i)	$H_0: \rho = 0$	A1	
		(ii)	$H_1: \rho < 0$	A1	
		(iii)	$p\text{-value} = 0.009830306$	(A1) for correct value	
			$p\text{-value} = 0.00983$	A1	
		(iv)	The null hypothesis is rejected.	A1	
			As $p\text{-value} < 0.05$.	R1	
					[6]
	(d)	(i)	$a = -1.533333333$		
			$a = -1.53$	A1	
			$b = 510.7333333$		
			$b = 511$	A1	
		(ii)	a represents the average increase of the maximum walking speed of a crab when its weight is increased by 1 gram.	A1	
					[3]

(e)	(i)	$H_0: \mu = 300$	A1
	(ii)	$H_1: \mu \neq 300$	A1
	(iii)	$z = -1.16$	A1
	(iv)	$p\text{-value} = 0.2452782275$	(A1) for correct value
		$p\text{-value} = 0.245$	A1
	(v)	The null hypothesis is not rejected.	A1
		As $p\text{-value} > 0.1$.	R1

[7]

AI HL Practice Set 3 Paper 1 Solution

1. (a) $260-100=(31-11)d$ (M1) for valid approach

$160=20d$

$d=8$

Thus, the common difference is 8. A1

[2]

(b) $u_{11}=100$

$\therefore u_1+(11-1)(8)=100$ (A1) for correct equation

$u_1=20$ A1

[2]

(c) S_{51}

$=\frac{51}{2}\left[2(20)+(51-1)(8)\right]$ (A1) for substitution

$=11220$ A1

[2]

2. (a) 4 A1

[1]

(b) The inter-quartile range

$=6-2.5$ (M1) for valid approach

$=3.5$ A1

[2]

(c) The required probability

$=\frac{8}{12}$ (M1) for valid approach

$=\frac{2}{3}$ A1

[2]

3. (a) $\cos \mathrm{A\hat{C}B} = \dfrac{\mathrm{AC}^2 + \mathrm{BC}^2 - \mathrm{AB}^2}{2(\mathrm{AC})(\mathrm{BC})}$ (M1) for cosine rule

$\cos \mathrm{A\hat{C}B} = \dfrac{54^2 + 54^2 - 35^2}{2(54)(54)}$ (A1) for substitution

$\cos \mathrm{A\hat{C}B} = 0.789951989$

$\mathrm{A\hat{C}B} = 37.81897498^\circ$

$\mathrm{A\hat{C}B} = 37.8^\circ$ A1

[3]

(b) The required area

$= \dfrac{1}{2}(\mathrm{AC})(\mathrm{BC})\sin \mathrm{A\hat{C}B}$ (M1) for area formula

$= \dfrac{1}{2}(54)(54)\sin 37.81897498^\circ$ (A1) for substitution

$= 893.999965 \text{ cm}^2$

$= 894 \text{ cm}^2$ A1

[3]

4. (a) $\mathrm{P}(X = 1) + \mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4) = 1$ M1

$\therefore 5k^2 + (k^2 + 6k) + (k^2 + k) + k^2 = 1$ A1

$8k^2 + 7k - 1 = 0$

$(k + 1)(8k - 1) = 0$ A1

$k = -1$ (*Rejected*) or $k = \dfrac{1}{8}$ AG

[3]

(b) $\mathrm{P}(X = 2 \mid X \le 2)$

$= \dfrac{\mathrm{P}(X = 2 \cap X \le 2)}{\mathrm{P}(X \le 2)}$

$= \dfrac{\mathrm{P}(X = 2)}{\mathrm{P}(X \le 2)}$ (M1) for valid approach

$= \dfrac{\left(\frac{1}{8}\right)^2 + 6\left(\frac{1}{8}\right)}{5\left(\frac{1}{8}\right)^2 + \left(\left(\frac{1}{8}\right)^2 + 6\left(\frac{1}{8}\right)\right)}$ (A1) for substitution

$= \dfrac{49}{54}$ A1

[3]

5. (a) $y = 5$ A1

[1]

(b) (i) $\left(5, \frac{7}{2}\right)$ A1

(ii) $k(5) + 2\left(\frac{7}{2}\right) - 47 = 0$ (M1) for substitution

$5k = 40$

$k = 8$ A1

(iii) $8x + 2(5) - 47 = 0$ (M1) for substitution

$8x = 37$

$x = \frac{37}{8}$

Thus, the required coordinates are

$\left(\frac{37}{8}, 5\right)$. A1

[5]

6. (a) $y = \frac{8}{7}$ A2

[2]

(c) $\left\{y : y \neq \frac{8}{7}, y \in \mathbb{R}\right\}$ A1

[1]

(d) $f(x) > g(x)$

$\frac{1-8x}{2-7x} > \frac{1}{2}x^2$

$\frac{1-8x}{2-7x} - \frac{1}{2}x^2 > 0$ M1

By considering the graph of $y = \frac{1-8x}{2-7x} - \frac{1}{2}x^2$,

$-1.439727 < x < 0.1239131$ or $\frac{2}{7} < x < 1.6015283$.

$\therefore -1.44 < x < 0.124$ or $\frac{2}{7} < x < 1.60$ A2

[3]

7. (a) Let $r\%$ be the nominal annual interest rate compounded yearly.

$(1+r\%)^6 = \left(1+\frac{9}{(100)(12)}\right)^{(12)(6)}$ (A1) for substitution

$1+r\% = 1.0075^{12}$

$r = 9.380689767$ (A1) for correct value

The real interest rate per year

$= 9.380689767\% - i\%$

$= (9.38069 - i)\%$ A1

[3]

(b) $89000\left(1+\frac{9.38069-i}{100}\right)^6 = 118000$ (M1) for setting equation

$89000\left(1+\frac{9.38069-i}{100}\right)^6 - 118000 = 0$ (A1) for correct approach

By considering the graph of

$y = 89000\left(1+\frac{9.38069-i}{100}\right)^6 - 118000$,

$i = 4.5676461$.

Thus, $i = 4.57$. A1

[3]

8. (a) The volume

$= \pi r^2 h$

$= \pi(4)^2(15)$ (A1) for substitution

$= 240\pi \text{ cm}^3$ A1

[2]

(b) The total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(15)$ (A1) for substitution

$= 152\pi \text{ cm}^2$ A1

[2]

(c) 26 A1

[1]

9. (a) $f'(x)$

$= 0 + 9(2x) + 2(3x^2)$ (A1) for correct approach

$= 18x + 6x^2$ A1

[2]

(b) $f'(x) = 0$

$18x + 6x^2 = 0$

$6x(3 + x) = 0$ (A1) for factorization

$x = 0$ or $x = -3$ A1

[2]

(c) (i) $f''(x) = 18 + 12x$ A1

(ii) $f''(-3)$

$= 18 + 12(-3)$

$= -18 < 0$ R1

Therefore, f attains its local maximum at $x = -3$.

Thus, the x-coordinate of the local maximum of f is -3. A1

(iii) 57 A1

[4]

10. (a) H_0: The data follows a Poisson distribution with mean 3. A1

[1]

(b) 36.9 A1

[1]

(c) 5 A1

[1]

(c) 26.3 A2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 11.070$. R1

[2]

11. (a) $\log\frac{1}{8}+\log\frac{1}{125}$

$=\log\left(\frac{1}{8}\cdot\frac{1}{125}\right)$ (A1) for correct formula

$=\log\frac{1}{1000}$

$=\log 10^{-3}$ (A1) for valid approach

$=-3$ A1

[3]

(b) $\ln e^{\frac{10}{3}}-\ln\sqrt[6]{e}$

$=\ln\frac{e^{\frac{10}{3}}}{e^{\frac{1}{6}}}$ (A1) for correct formula

$=\ln e^{\frac{10}{3}-\frac{1}{6}}$

$=\ln e^{\frac{19}{6}}$ (A1) for valid approach

$=\frac{19}{6}$ A1

[3]

12. (a) An unbiased estimate

$=\frac{700+698+\cdots+641}{12}$ (A1) for correct approach

$=663\text{ g}$ A1

[2]

(b) s_{n-1}

$=\sqrt{\frac{(700-663)^2+(698-663)^2+\cdots+(641-663)^2}{12-1}}$ (A1) for correct approach

$=31.53353194\text{ g}$

$=31.5\text{ g}$ A1

[2]

(c) 99% confidence interval:

$(634.73, 691.27)$ A2

[2]

13. (a) $g(x) = -f(x)$ (M1) for valid approach

$g(x) = -((x+1)^2 + 3)$

$g(x) = -(x+1)^2 - 3$ A1

[2]

(b) (i) $1 - p = -10$ (M1) for translation

$p = 11$ A1

(ii) $-3 + q = 0$ (M1) for translation

$q = 3$ A1

[4]

14. (a) $X \sim \text{Po}(1.75)$

$\text{P}(X \geq 3)$

$= 1 - \text{P}(X \leq 2)$ (M1) for valid approach

$= 1 - 0.7439696955$

$= 0.2560303045$

$= 0.256$ A1

[2]

(b) $Y \sim \text{Po}(12.25)$ (M1) for valid approach

$\text{P}(Y \leq 14)$

$= 0.7489477707$

$= 0.749$ A1

[2]

(c) The required probability

$= \text{P}(X \leq 2)^7$ (M1) for valid approach

$= 0.7439696955^7$

$= 0.1261498443$

$= 0.126$ A1

[2]

15. (a) By considering the graph of

$y = -x^3 + 17x^2 - 86x + 112$, $x = 2$, $x = 7$ or $x = 8$. (M1) for valid approach

Thus, the y-intercepts are 2, 7 and 8. A2

[3]

(b) The total area of the region

$= \int_2^8 \left| -y^3 + 17y^2 - 86y + 112 \right| \mathrm{d}y$ (A1) for correct approach

$= 73.83333519$

$= 73.8$ A1

[2]

16. (a) (i) 152.6 A1

(ii) 150.6 A1

(iii) 168.3 A1

[3]

(b) SS_{res}

$= (33\sqrt{24} - 160)^2 + (33\sqrt{26} - 160)^2$ (A1) for correct approach

$+ (33\sqrt{28} - 173)^2$

$= 73.75362941$

$= 73.8$ A1

[2]

(c) Model 2 A1

[1]

17. (a) $\mathbf{A}$

$= (\mathbf{A}^{-1})^{-1}$ (M1) for valid approach

$= \begin{pmatrix} 2 & 1 & 1 \\ 2 & -3 & -5 \\ -1 & 1 & 3 \end{pmatrix}$

$\therefore p = 2$, $q = 1$ A2

[3]

(b) $\begin{cases} 4x + 2y + 2z = 3 \\ x - 7y - 12z = 5 \\ x + 3y + 8z = 9 \end{cases}$ can be expressed as

$\begin{cases} 0.4x + 0.2y + 0.2z = 0.3 \\ 0.1x - 0.7y - 1.2z = 0.5 \\ 0.1x + 0.3y + 0.8z = 0.9 \end{cases}$. (M1) for valid approach

$\mathbf{A}^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.5 \\ 0.9 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A} \begin{pmatrix} 0.3 \\ 0.5 \\ 0.9 \end{pmatrix}$ M1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5.4 \\ 2.9 \end{pmatrix}$

$\therefore x = 2$, $y = -5.4$, $z = 2.9$ A3

[5]

18. (a) $\sin 5x + \cos 4x = 0$

By considering the graph of $y = \sin 5x + \cos 4x$,

$x = 0.5235988$ or $x = 1.2217305$.

$\therefore x = 0.524$ or $x = 1.22$ A2

[2]

(b) $\sin 10x + \cos 8x$ is formed by a horizontal compression of $\sin 5x + \cos 4x$ of scale factor 2. R1

Therefore, there are still two distinct real roots when the range of x is halved at the same time. R1

Thus, the statement is incorrect. A1

[3]

(c) 6 A1

[1]

AI HL Practice Set 3 Paper 2 Solution

1. (a) $a = 5.6$ A1

$b = 34.8$ A1

[2]

(b) The estimated hardness

$= 5.6(6.3) + 34.8$ (A1) for substitution

$= 70.08$ A1

[2]

(c) The required probability

$= \frac{120 - 56}{120}$ (M1) for valid approach

$= \frac{8}{15}$ A1

[2]

(d) (i) Let X be the number of selected ingots of the hardness at least 65, where

$X \sim \mathrm{B}\left(10, \frac{8}{15}\right)$.

The required probability

$= \mathrm{P}(X = 5)$ (M1) for valid approach

$= 0.2406733955$

$= 0.241$ A1

(ii) The required probability

$= \mathrm{P}(X < 4)$ (M1) for valid approach

$= 0.1226252054$

$= 0.123$ A1

(iii) $\frac{16}{3}$ A1

[5]

(d) (i) $H_1: \mu_1 \neq \mu_2$ A1

(ii) $p\text{-value} = 0.0741679182$ (A1) for correct value

$p\text{-value} = 0.0742$ A1

(iii) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. R1

[5]

2. (a) $P(0)=116$

$\therefore a+b\times c^0=116$ (M1) for setting equation

$a+b=116$ A1

[2]

(b) $P(1)=172$

$\therefore a+b\times c^{-1}=172$ (M1) for setting equation

$a+\frac{b}{c}=172$ A1

[2]

(c) (i) $\log_c 81=4$

$\therefore c^4=81$ M1

$c^4=3^4$ A1

$c=3$ AG

(ii) The system is $\begin{cases} a+b=116 \\ a+\frac{1}{3}b=172 \end{cases}$. (M1) for valid approach

Solving, we have $a=200$ and $b=-84$. A2

[5]

(d) The number of elephants

$=200-84\times 3^{-3}$ (M1) for substitution

$=196.8888889$

$=197$ A1

[2]

(e) 200 A1

[1]

(f) $200-84\times 3^{-t}>195$ (M1) for setting inequality

$5-84\times 3^{-t}>0$

By considering the graph of $y=5-84\times 3^{-t}$,

$t=2.5681297$.

Thus, the number of years needed is 2.57 years. A1

[2]

(g) By considering the graphs of $y = 200 - 84 \times 3^{-t}$, $y = 170$, $y = 180$ and $y = 190$, y reaches 170, 180 and 190 at $t_1 = 0.9372$, $t_2 = 1.3062702$ and $t_3 = 1.9372$ respectively. M1A1

$\because 2(t_2 - t_1)$

$= 2(1.3062702 - 0.9372)$

$= 0.7381404$

$\neq t_3 - t_2$ R1

Thus, the claim is disagreed. A1

[4]

3. (a) (i) $(4, 8)$ A2

(ii) $\{y : 4 \le y \le 8,\ y \in \mathbb{R}\}$ A2

[4]

(b) $f'(x)$

$= -0.25(2x) + 2(1) + 0$ (A1) for correct derivatives

$= -0.5x + 2$ A1

[2]

(c) $f'(x) = -1$ M1

$\therefore -0.5x + 2 = -1$ A1

$-0.5x = -3$

$x = 6$ A1

$f(6)$

$= -0.25(6)^2 + 2(6) + 4$ M1

$= 7$

Thus, the coordinates of P are $(6, 7)$. AG

[4]

(d) The equation of the tangent:

$y - 7 = -1(x - 6)$ (A1) for substitution

$y - 7 = -x + 6$

$x + y - 13 = 0$ A1

[2]

(e) (i) 4 A1

(ii) 5.75 A1

[2]

(f) The estimate of $\int_0^8 f(x)\mathrm{d}x$

$= \frac{1}{2}(1)\left[4 + 4 + 2\begin{pmatrix} 5.75 + 7 + 7.75 \\ +8 + 7.75 + 7 + 5.75 \end{pmatrix}\right]$ (A2) for substitution

$= 53$ A1

[3]

(g) Underestimate A1

[1]

4. (a) The period of W_2

$= \frac{2\pi}{2\pi}$ — (M1) for valid approach

$= 1\text{ s}$ — A1

[2]

(b) $W_1 + W_2$

$= 11\cos(2\pi t - 0.1) + 13\cos(2\pi t - 0.3)$

$= \text{Re}(11e^{(2\pi t - 0.1)\text{i}}) + \text{Re}(13e^{(2\pi t - 0.3)\text{i}})$ — (M1) for valid approach

$= \text{Re}(11e^{(2\pi t - 0.1)\text{i}} + 13e^{(2\pi t - 0.3)\text{i}})$ — (A1) for correct approach

$= \text{Re}(e^{2\pi t\text{i}}(11e^{-0.1\text{i}} + 13e^{-0.3\text{i}}))$

$\therefore z + w = 11e^{-0.1\text{i}} + 13e^{-0.3\text{i}}$ — A1

[3]

(c) (i) $z = 11e^{-0.1\text{i}}$

$z = 11(\cos(-0.1) + \text{i}\sin(-0.1))$ — A1

(ii) $w = 13e^{-0.3\text{i}}$

$w = 13(\cos(-0.3) + \text{i}\sin(-0.3))$ — A1

[2]

(d) (i) $z + w$

$= 11(\cos(-0.1) + \text{i}\sin(-0.1))$

$+ 13(\cos(-0.3) + \text{i}\sin(-0.3))$

$= (11\cos(-0.1) + 13\cos(-0.3))$

$+ \text{i}(11\sin(-0.1) + 13\sin(-0.3))$ — (M1) for valid approach

$z + w = 23.36442018 - 4.93993027\text{i}$ — (A1) for correct values

L

$= \sqrt{23.36442018^2 + (-4.93993027)^2}$ — M1

$= 23.88093468$

$= 23.9$ — A1

(ii) α

$= \tan^{-1}\frac{-4.93993027}{23.36442018}$ — M1

$= -0.2083610278$

$= -0.208$ — A1

[6]

(e) $W_1 + W_2$

$= \mathrm{Re}(e^{2\pi t\mathrm{i}}(z+w))$

$= \mathrm{Re}(e^{2\pi t\mathrm{i}} \cdot 23.88093468e^{-0.2083610278\mathrm{i}})$ (M1) for substitution

$= \mathrm{Re}(23.88093468e^{2\pi t\mathrm{i}-0.2083610278\mathrm{i}})$ (A1) for correct approach

$= 23.88093468\cos(2\pi t - 0.2083610278)$

$= 23.9\cos(2\pi t - 0.208)$ A1

[3]

5. (a) Eulerian trail does not exist. A1
As there are more than two vertices of odd degrees. A1

[2]

(b)
$$\mathbf{M}=\begin{pmatrix}0&1&0&0&0&1&1\\1&0&1&0&0&0&1\\0&1&0&1&0&0&1\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\\1&0&0&0&1&0&1\\1&1&1&1&1&1&0\end{pmatrix}$$
A2

[2]

(c)
$$\mathbf{M}^4=\begin{pmatrix}24&18&23&18&23&18&32\\18&24&18&23&18&23&32\\23&18&24&18&23&18&32\\18&23&18&24&18&23&32\\23&18&23&18&24&18&32\\18&23&18&23&18&24&32\\32&32&32&32&32&32&60\end{pmatrix}$$
(M1) for valid approach

Thus, the total number of walks of length 4 from D to A is 18. A1

[2]

(d) For any three edges correct A1
For all edges correct A1

1. Choose AF of weight 50
2. Choose BC of weight 52
3. Choose AG of weight 53
4. Choose DE of weight 54
5. Choose CG of weight 58
6. Choose EF of weight 59

Thus, the minimum spanning tree is a tree containing AF, BC, AG, DE, CG and EF. A1

[3]

(e) 326 A1

[1]

(f) For all edges correct A2

1. Choose ED of weight 54
2. Choose DC of weight 61
3. Choose CB of weight 52
4. Choose BA of weight 63
5. Choose AF of weight 50
6. Choose FG of weight 57
7. Choose GE of weight 61

Thus, an upper bound of the total weight of a cycle that passes through all seven vertices is 398. AG

[2]

(g) For any three edges correct A1

For all edges correct A1

1. Choose AF of weight 50
2. Choose BC of weight 52
3. Choose AG of weight 53
4. Choose CG of weight 58
5. Choose CD of weight 61

Therefore, the weight of a minimum spanning tree after deleting the vertex E is 274. A1

The required lower bound

$= 274 + 54 + 59$

$= 387$ A1

[4]

6. (a) (i) $\overrightarrow{\mathrm{BD}}$

$$= \begin{pmatrix} 0 \\ -\pi \\ 0 \end{pmatrix} - \begin{pmatrix} \pi \\ 0 \\ \pi \end{pmatrix}$$

$$= \begin{pmatrix} -\pi \\ -\pi \\ -\pi \end{pmatrix}$$ A1

The vector equation of BD:

$$\mathbf{r} = \begin{pmatrix} \pi \\ 0 \\ \pi \end{pmatrix} + t\begin{pmatrix} -\pi \\ -\pi \\ -\pi \end{pmatrix}$$ A1

(ii) $$\begin{cases} x = \pi - \pi t \\ y = -\pi t \\ z = \pi - \pi t \end{cases}$$ A1

$$\overrightarrow{\mathrm{CE}} = \begin{pmatrix} \pi - \pi t \\ -\pi t \\ \pi - \pi t \end{pmatrix} - \begin{pmatrix} \pi \\ 0 \\ 0 \end{pmatrix}$$

$$\overrightarrow{\mathrm{CE}} = \begin{pmatrix} -\pi t \\ -\pi t \\ \pi - \pi t \end{pmatrix}$$ A1

(iii) $\overrightarrow{CE} \cdot \overrightarrow{BD} = 0$

$\therefore (-\pi t)(-\pi) + (-\pi t)(-\pi)$ $+(\pi - \pi t)(-\pi) = 0$ M1

$\pi^2 t + \pi^2 t - \pi^2 + \pi^2 t = 0$

$3\pi^2 t = \pi^2$

$t = \frac{1}{3}$ A1

$$\therefore \begin{cases} x = \pi - \pi\left(\frac{1}{3}\right) = \frac{2}{3}\pi \\ y = -\pi\left(\frac{1}{3}\right) = -\frac{1}{3}\pi \\ z = \pi - \pi\left(\frac{1}{3}\right) = \frac{2}{3}\pi \end{cases}$$ M1

Therefore, the coordinates of E are $\left(\frac{2}{3}\pi, -\frac{1}{3}\pi, \frac{2}{3}\pi\right)$. AG

[7]

(b) (i) $\overrightarrow{BA} = \begin{pmatrix} -\pi \\ 0 \\ 0 \end{pmatrix}$ A1

(ii) $\mathbf{w}$

$= \overrightarrow{BA} \times \overrightarrow{BD}$

$= \begin{pmatrix} -\pi \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -\pi \\ -\pi \\ -\pi \end{pmatrix}$

$= \begin{pmatrix} (0)(-\pi) - (0)(-\pi) \\ (0)(-\pi) - (-\pi)(-\pi) \\ (-\pi)(-\pi) - (0)(-\pi) \end{pmatrix}$ (A1) for substitution

$= \begin{pmatrix} 0 \\ -\pi^2 \\ \pi^2 \end{pmatrix}$ A1

(iii) $\begin{pmatrix} 0 \\ -\pi^2 \\ \pi^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \left|\begin{pmatrix} 0 \\ -\pi^2 \\ \pi^2 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\right| \cos\theta$ (M1) for valid approach

$(0)(1)+(-\pi^2)(1)+(\pi^2)(2)$
$=(\sqrt{0^2+(-\pi^2)^2+(\pi^2)^2})(\sqrt{1^2+1^2+2^2})\cos\theta$ A1

$\pi^2=\sqrt{12\pi^4}\cos\theta$ (A1) for correct approach

$\cos\theta=\dfrac{1}{\sqrt{12}}$

$\theta=1.277953555\text{ rad}$

Therefore, the required acute angle is
1.28 rad. A1

[7]

7. (a) $\begin{cases} \dfrac{dv}{dt} = 7v - 10x \\ \dfrac{dx}{dt} = v \end{cases}$ A1

[1]

(b) $\det(\mathbf{M} - \lambda \mathbf{I})$

$= \begin{vmatrix} 7-\lambda & -10 \\ 1 & 0-\lambda \end{vmatrix}$ (M1) for valid approach

$= (7-\lambda)(-\lambda) - (-10)(1)$

$= -7\lambda + \lambda^2 + 10$

$= \lambda^2 - 7\lambda + 10$ A1

[2]

(c) $\lambda_1 = 2$, $\lambda_2 = 5$ A2

[2]

(d) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$ A2

[2]

(e) $\mathbf{X} = Ae^{2t}\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} + Be^{5t}\begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} 3 \\ 0 \end{pmatrix} = Ae^{2(0)}\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} + Be^{5(0)}\begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$ (M1) for substitution

$\begin{cases} 3 = A + B \\ 0 = \frac{1}{2}A + \frac{1}{5}B \end{cases}$

By solving this system, $A = -2$ and $B = 5$. (A1) for correct values

$\therefore v = -2e^{2t} + 5e^{5t}$ and $x = -e^{2t} + e^{5t}$. A2

[5]

AI HL Practice Set 3 Paper 3 Solution

1. (a) (i) 42.3 s A1

(ii) 1.47 s^2 A1

[2]

(b) $\text{P}(P_1 + P_2 + P_3 < 40.5)$

$= 0.0688229545$ (A1) for correct value

$= 0.0688$ A1

[2]

(c) (i) The required variance

$= \dfrac{0.7^2}{5}$ (A1) for correct approach

$= 0.098\text{ s}^2$ A1

(ii) The required variance

$= \dfrac{0.55^2}{5}$ (A1) for correct approach

$= 0.0605\text{ s}^2$ A1

[4]

(d) (i) $\bar{P} - \bar{Q} \sim \text{N}\left(14.1 - 14.9, \dfrac{0.7^2}{5} + \dfrac{0.55^2}{5}\right)$ A2

$\text{P}(\bar{P} - \bar{Q} > 0)$ M1

$= 0.0222451001$ (A1) for correct value

$= 0.0222$ A1

(ii) $\text{P}(-0.2 < \bar{P} - \bar{Q} < 0.2)$ M1

$= 0.0598891201$ (A1) for correct value

$= 0.0599$ A1

[8]

(e) (i) An unbiased estimate

$= \frac{13.9+14.7+13.5+14.0+14.2}{5}$ (A1) for correct approach

$= 14.06$ s A1

(ii) s_{n-1}

$= \sqrt{\frac{(13.9-14.06)^2+(14.7-14.06)^2 + \cdots + (14.2-14.06)^2}{5-1}}$ (A1) for correct approach

$= 0.4393176527$ s

$= 0.439$ s A1

[4]

(f) 95% confidence interval:

$(13.515, 14.605)$ A2

[2]

(g) (i) $H_0: \mu_d = 0$ A1

(ii) $H_1: \mu_d > 0$ A1

(iii) $p\text{-value} = 0.7875667907$ (A1) for correct value

$p\text{-value} = 0.788$ A1

(iv) –0.868 A1

(v) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. R1

[7]

2. (a) (i) $(10, -10)$ A1

(ii) 50 A1

[2]

(b) (i) $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$

$= A_3\begin{pmatrix} 10 \\ -10 \end{pmatrix}$ (M1) for substitution

$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^3 \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ A1

$= \begin{pmatrix} 80 \\ -80 \end{pmatrix}$ A1

(ii) The required area

$= \dfrac{(80)(80)}{2}$ M1

$= 3200$ A1

(iii) $\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^n \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ M1A1

$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 2^n & 0 \\ 0 & 2^n \end{pmatrix}\begin{pmatrix} 10 \\ -10 \end{pmatrix}$ A1

$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 10 \cdot 2^n \\ -10 \cdot 2^n \end{pmatrix}$ A1

$\therefore x_n - y_n$

$= 10 \cdot 2^n - (-10 \cdot 2^n)$

$= 20 \cdot 2^n$ A1

$= 5 \cdot 2^2 \cdot 2^n$ M1

$= 5 \cdot 2^{n+2}$ AG

[11]

(c) (i) $\begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$

$= A_4\begin{pmatrix} 0 \\ -10 \end{pmatrix}$ (M1) for substitution

$$=\begin{pmatrix}1.1^4 & 0\\ 0 & 1.1^4\end{pmatrix}\begin{pmatrix}1.1^3 & 0\\ 0 & 1.1^3\end{pmatrix}\begin{pmatrix}1.1^2 & 0\\ 0 & 1.1^2\end{pmatrix}$$

$$\begin{pmatrix}1.1^1 & 0\\ 0 & 1.1^1\end{pmatrix}\begin{pmatrix}0\\ -10\end{pmatrix}$$ A1

$$=\begin{pmatrix}1.1^{10} & 0\\ 0 & 1.1^{10}\end{pmatrix}\begin{pmatrix}0\\ -10\end{pmatrix}$$ M1A1

$$=\begin{pmatrix}0\\ -10\cdot 1.1^{10}\end{pmatrix}$$ A1

(ii) $y_n = -10\cdot 1.1^{1+2+\cdots+n}$ A1

$y_n < -375$

$\therefore -10\cdot 1.1^{1+2+\cdots+n} < -375$ (M1) for setting inequality

$1.1^{1+2+\cdots+n} > 37.5$

$1.1^{1+2+\cdots+n} - 37.5 > 0$

$1.1^{\frac{n}{2}(n+1)} - 37.5 > 0$ (A1) for correct inequality

By considering the graph of

$y = 1.1^{\frac{n}{2}(n+1)} - 37.5$, $n > 8.2351929$. (A1) for correct value

Thus, the least value of n is 9. A1

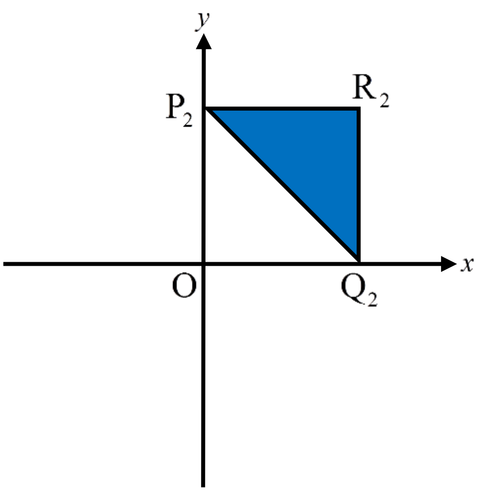
[10]

(d) (i) $\begin{pmatrix}\cos 180^\circ & \sin 180^\circ\\ \sin 180^\circ & -\cos 180^\circ\end{pmatrix}\begin{pmatrix}\cos 90^\circ & \sin 90^\circ\\ \sin 90^\circ & -\cos 90^\circ\end{pmatrix}$ A1

(ii) For correct quadrant A1

For correct points A1

[3]



AI HL Practice Set 4 Paper 1 Solution

1. (a) $V = \frac{1}{3}\pi r^2 h$

$\therefore 128\pi = \frac{1}{3}\pi r^2 (6)$ (A1) for correct equation

$r^2 = 64$

$r = 8$

Thus, the required radius is 8 cm. A1

[2]

(b) l

$= \sqrt{r^2 + h^2}$ (M1) for valid approach

$= \sqrt{8^2 + 6^2}$

$= 10$

Thus, the required slant height is 10 cm. A1

[2]

(c) The total surface area

$= \pi r^2 + \pi r l$

$= \pi(8)^2 + \pi(8)(10)$ (A1) for substitution

$= 144\pi\text{ cm}^2$ A1

[2]

2. (a) (i) 20 hours A1

(ii) 15 hours A1

[2]

(b) 5 workers worked for more than 30 hours. Therefore, 12.5% of the workers worked for more than 30 hours. (R1) for correct argument

$\therefore k = 30$ A1

[2]

3.	(a)	(i)	c_n	A1	
		(ii)	b_n	A1	
					[2]
	(b)	(i)	1.25	A1	
		(ii)	$\frac{3125}{128}$	A1	
		(iii)	S_8		
			$=\frac{10(1.25^8-1)}{1.25-1}$	(A1) for substitution	
			$=198.4185791$		
			$=198$	A1	
					[4]
4.	(a)	(i)	The radius		
			$=\sqrt{(10-6)^2+(12-14)^2}$	(A1) for substitution	
			$=4.472135955\text{ km}$		
			$=4.47$ km	A1	
		(ii)	4 km	A1	
		(iii)	The apartment at P	A1	
					[4]
	(b)		$x+y-20=0$	A2	
					[2]

5. (a) $\mathrm{E}(X)=8.64$

$\therefore 0.72n=8.64$ — (A1) for correct equation

$n=12$ — A1

[2]

(b) $\mathrm{Var}(X)$

$=(12)(0.72)(1-0.72)$ — (A1) for substitution

$=2.4192$ — A1

[2]

(c) $\mathrm{P}(X\geq 11)$

$=1-\mathrm{P}(X\leq 10)$ — (A1) for substitution

$=0.1099809898$

$=0.110$ — A1

[2]

6. (a) By TVM Solver:

$\mathrm{N}=120$
$\mathrm{I\%}=4.5$
$\mathrm{PV}=0$
$\mathrm{PMT}=-200$
$\mathrm{FV}=?$
$\mathrm{P/Y}=12$
$\mathrm{C/Y}=1$
$\mathrm{PMT:END}$

(A2) for correct values

$\mathrm{FV}=30095.13482$

Thus, the value of the investment after ten years is $\$30100$. — A1

[3]

(b) By TVM Solver:

$\mathrm{N}=144$
$\mathrm{I\%}=4.5$
$\mathrm{PV}=0$
$\mathrm{PMT}=?$
$\mathrm{FV}=5\times 30095.13482$
$\mathrm{P/Y}=12$
$\mathrm{C/Y}=1$
$\mathrm{PMT:END}$

(A2) for correct values

$\mathrm{PMT}=-794.6316652$

Thus, the new amount of deposit is $\$795$. — A1

[3]

7. (a) x

$= -\frac{b}{2a}$

$= -\frac{100}{2(-1)}$ (A1) for substitution

$= 50$ A1

[2]

(b) The required maximum height

$= -50^2 + 100(50) - 1600$ A1

$= -2500 + 5000 - 1600$

$= 900 \text{ m}$ AG

[1]

(c) $V = 0$

$-x^2 + 100x - 1600 = 0$

$x = 20$ or $x = 80$ (A1) for correct values

The required horizontal distance

$= 80 - 20$ (M1) for valid approach

$= 60 \text{ m}$ A1

[3]

8. (a) $\frac{\sin \mathrm{A\hat{C}B}}{\mathrm{AB}} = \frac{\sin \mathrm{A\hat{B}C}}{\mathrm{AC}}$ (M1) for sine rule

$\frac{\sin \mathrm{A\hat{C}B}}{13.9} = \frac{\sin 60.8^\circ}{17.7}$ (A1) for substitution

$\mathrm{A\hat{C}B} = 43.27612856^\circ$

$\mathrm{A\hat{C}B} = 43.3^\circ$ A1

[3]

(b) The area of the triangle ABC

$= \frac{1}{2}(\mathrm{AB})(\mathrm{AC})\sin \mathrm{B\hat{A}C}$ (M1) for area formula

$= \frac{1}{2}(13.9)(17.7)\sin(180^\circ - 60.8^\circ - 43.27612856^\circ)$ (A1) for substitution

$= 119.3212815 \text{ cm}^2$

$= 119 \text{ cm}^2$ A1

[3]

9. (a) $\frac{dx}{dt} = \pi x^2 \cos \pi t$

$\frac{1}{x^2}dx = \pi \cos \pi t dt$ (M1) for valid approach

$\therefore \int \frac{1}{x^2}dx = \int \pi \cos \pi t dt$ A1

[2]

(b) Let $u = \pi t$.

$\frac{du}{dt} = \pi \Rightarrow du = \pi dt$ A1

$\therefore \int \frac{1}{x^2}dx = \int \cos u du$ (A1) for correct working

$-\frac{1}{x} = \sin u + C$

$\frac{1}{x} = -\sin \pi t + C$ A1

[3]

(c) $\frac{1}{1} = -\sin 2.5\pi + C$ (M1) for substitution

$1 = -1 + C$

$C = 2$ (A1) for correct value

$\therefore \frac{1}{x} = -\sin \pi t + 2$

$x = \frac{1}{-\sin \pi t + 2}$ A1

[3]

10. (a) An unbiased estimate

$= \bar{X}$ (A1) for correct approach

$= \frac{18.95 + 25.15}{2}$

$= 22.05$ A1

[2]

(b) $25.15 - 18.95 = 2(1.959963986)\left(\frac{\sigma}{\sqrt{10}}\right)$ M1A1

$\sigma = 5.001653508$

$\sigma = 5.00$ A1

[3]

11. (a) $y=\sqrt{3-x}$

$\Rightarrow x=\sqrt{3-y}$ — (M1) for swapping variables

$10=\sqrt{3-y}$

$100=3-y$

$y=-97$ — (M1) for valid approach

$\therefore f^{-1}(10)=-97$ — A1

[3]

(b) (i) 5 — A1

(ii) $(f^{-1}\circ g^{-1})(\pi)$

$= f^{-1}(5)$

$5=\sqrt{3-x}$ — (M1) for valid approach

$25=3-x$

$x=-22$

$\therefore f^{-1}(5)=-22$ — A1

[3]

12. (a) (i) $a=32$ — A1

$b=20.6$ — A1

(ii) The estimated number of oil refills

$=32(2.5)+20.6$ — (A1) for substitution

$=100.6$ — A1

[4]

(b) (i) $r=0.9765724246$

$r=0.977$ — A1

(ii) $R^2=0.9536937004$

$R^2=0.954$ — A1

(iii) 95.4% of the variability of the data is explained by the regression model. — A1

[3]

13.	(a)	CE	A1	
				[1]
	(b)	For any two edges correct	A1	
		For all edges correct	A1	
		1. Choose BE of weight 22		
		2. Choose DE of weight 24		
		3. Choose AD of weight 10		
		4. Choose AC of weight 20		
		Thus, the minimum spanning tree is a tree containing BE, DE, AD and AC.	A1	
				[3]
	(c)	76	A1	
				[1]
14.	(a)	(i) $H_0:\ p = 0.25$	A1	
		(ii) $H_1:\ p > 0.25$	A1	
				[2]
	(b)	$\mathrm{P}(X \geq 39) = 0.4193193762$	(M1) for valid approach	
		Thus, the p-value is 0.419.	A1	
				[2]
	(c)	The null hypothesis is not rejected.	A1	
		As $p\text{-value} > 0.05$.	R1	
				[2]

15. (a) $y = e^{5x}$

$\Rightarrow x = e^{5y}$ (M1) for swapping variables

$5y = \ln x$

$y = \frac{1}{5}\ln x$ (A1) for changing subject

$\therefore f^{-1}(x) = \frac{1}{5}\ln x$ A1

[3]

(b) $\{y : y \in \mathbb{R}\}$ A1

[1]

(c) $(g \circ f)(x)$

$= g(f(x))$

$= (3 + \ln f(x))^2$

$= (3 + \ln e^{5x})^2$ (M1) for substitution

$= (3 + 5x)^2$ (A1) for correct approach

$= 25x^2 + 30x + 9$ A1

[3]

16. (a) Rotation anticlockwise of $\frac{5\pi}{6}$ radians about the origin. A1

[1]

(b) $\begin{pmatrix} 8 \\ 0 \end{pmatrix} = \mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix}$ (M1) for valid approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{T}^{-1}\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6.92820323 \\ -4 \end{pmatrix}$

Thus, the coordinates of P are $(-6.93, -4)$. A1

[3]

(c) 12 A2

[2]

17. (a) $f'(x)$

$= \left(\frac{1}{x^2+4}\right)(2x)$ (M1) for chain rule

$= \frac{2x}{x^2+4}$ A1

[2]

(b) $\frac{6}{13}$ A1

[1]

(c) $13x + my = 39 + m\ln 13$

$my = -13x + 39 + m\ln 13$

$y = -\frac{13}{m}x + \frac{39 + m\ln 13}{m}$ (M1) for valid approach

$\therefore -\frac{13}{m} \times \frac{6}{13} = -1$ (A1) for correct equation

$m = 6$

$13x + 6(0) = 39 + 6\ln 13$ (M1) for substitution

$x = 3 + \frac{6}{13}\ln 13$

Thus, the x-intercept of the normal is

$x = 3 + \frac{6}{13}\ln 13$. A1

[4]

18. (a) By considering the graph of $y = \det(\mathbf{T} - \lambda\mathbf{I})$,

$\lambda = 0.42$ or $\lambda = 1$. (M1) for valid approach

$\therefore \lambda_1 = 0.42$, $\lambda_2 = 1$ A2

[3]

(b) $\mathbf{v}_{10}$

$= \begin{pmatrix} 0.73 & 0.31 \\ 0.27 & 0.69 \end{pmatrix}^{10} \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$ (M1) for valid approach

$= \begin{pmatrix} 0.5344597887 \\ 0.4655402113 \end{pmatrix}$

$= \begin{pmatrix} 0.534 \\ 0.466 \end{pmatrix}$ A1

[2]

(c) $\mathbf{v}$ is the eigenvector of $\mathbf{T}$ corresponding to

$\lambda_2 = 1$. (R1) for correct reasoning

$\therefore \mathbf{v} = \begin{pmatrix} \frac{31}{58} \\ \frac{27}{58} \end{pmatrix}$ A1

[2]

AI HL Practice Set 4 Paper 2 Solution

1. (a) The gradient of L_1

$=\frac{40-0}{0-30}$ (A1) for substitution

$=-\frac{4}{3}$ A1

[2]

(b) The equation of L_1:

$y-40=-\frac{4}{3}(x-0)$ (A1) for substitution

$3y-120=-4x$

$4x+3y-120=0$ A1

[2]

(c) The gradient of L_2

$=-1\div-\frac{4}{3}$

$=\frac{3}{4}$ (A1) for correct value

The equation of L_2:

$y=\frac{3}{4}x$ A1

[2]

(d) $4x+3\left(\frac{3}{4}x\right)-120=0$ (M1) for substitution

$6.25x=120$

$x=19.2$

$y=\frac{3}{4}(19.2)$ (M1) for substitution

$y=14.4$

Thus, the coordinates of C are $(19.2, 14.4)$. A1

[3]

(e) The area of the triangle OBC

$=\frac{(40-0)(19.2-0)}{2}$ (M1) for valid approach

$=384$ A1

[2]

(f) $BC = \sqrt{(0-19.2)^2 + (40-14.4)^2}$ (A1) for substitution

$BC = 32$ (A1) for correct value

$OC = \sqrt{(19.2-0)^2 + (14.4-0)^2}$

$OC = 24$ (A1) for correct value

The perimeter of the triangle OBC

$= 24 + 32 + 40$

$= 96$ A1

[4]

(g) $\frac{3}{4}k$ A1

[1]

(h) $\frac{(BC)(CD)}{2} = 624$ (A1) for correct equation

$32CD = 1248$

$CD = 39$ (A1) for correct value

$\therefore \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} = 39$ (A1) for correct equation

$\sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39 = 0$

By considering the graph of

$y = \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39$, $k = -12$ or

$k = 50.4$ (*Rejected*).

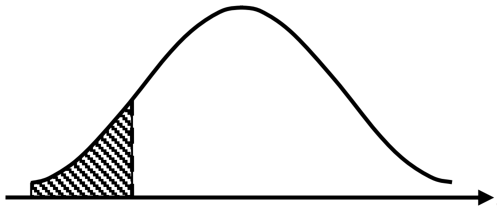
$\therefore k = -12$ A1

[4]

2. (a) For vertical line clearly to the left of the mean A1

For shading to the left of the vertical line A1

[2]



(b) (i) Let X be the volume of a randomly selected milk soda.

The required probability

$= \text{P}(X < 490)$ (M1) for valid approach

$= 0.105649839$

$= 0.106$ A1

(ii) The required probability

$= \text{P}(X > 483 \mid X < 490)$ (M1) for valid approach

$= \dfrac{\text{P}(X > 483 \cap X < 490)}{\text{P}(X < 490)}$

$= \dfrac{\text{P}(483 < X < 490)}{\text{P}(X < 490)}$ (A1) for correct approach

$= 0.8410480651$

$= 0.841$ A1

[5]

(c) The required probability

$= 2 \times \text{P}(X < 490) \times (1 - \text{P}(X < 490))$ (M1) for valid approach

$= 2 \times 0.105649839 \times (1 - 0.105649839)$ (A1) for substitution

$= 0.188975901$

$= 0.189$ A1

[3]

(d) (i) 0.327 A2

(ii) 0.0803 A2

(iii) –$1.29 A2

[6]

3.	(a)	(i)	$(6.67, 50.8)$	A2
		(ii)	$2 < x < 6.67$	A2
				[4]
	(b)	(i)	$f'(x) = -3x^2 + 13(2x) - 40(1) + 0$	(A1) for correct derivatives
			$f'(x) = -3x^2 + 26x - 40$	A1
		(ii)	15	A1
		(iii)	The equation of the tangent:	
			$y - f(5) = 15(x-5)$	M1A1
			$y - 36 = 15x - 75$	A1
			$15x - y - 39 = 0$	AG
				[6]
	(c)	(i)	9	A1
		(ii)	$\int_2^9 f(x)\mathrm{d}x$	A1
		(iii)	$\int_2^9 f(x)\mathrm{d}x = \frac{2401}{12}$	A2
				[4]
	(d)	The estimate of $\int_2^9 f(x)\mathrm{d}x$		
		$= \frac{1}{2}(1.75)\begin{bmatrix} f(2) + f(9) \\ +2(f(3.75) + f(5.5) + f(7.25)) \end{bmatrix}$		(A2) for substitution
		$= \frac{1}{2}(1.75)\left[0 + 0 + 2\begin{pmatrix} 16.078125 \\ +42.875 + 48.234375 \end{pmatrix}\right]$		(A1) for correct approach
		$= 187.578125$		
		$= 188$		A1
				[4]
	(e)	Underestimate		A1
				[1]

4. (a) The required distance

$= \sqrt{(12-0)^2 + (5-0)^2}$ (A1) for substitution

$= 13$ A1

[2]

(b) $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ A2

[2]

(c) The velocity vector

$= \frac{1}{2}\left(2\begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 12 \\ 5 \end{pmatrix}\right)$ (M1) for valid approach

$= \frac{1}{2}\begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 2.5 \end{pmatrix}$ A1

[2]

(d) $\begin{pmatrix} 8 \\ 10 \end{pmatrix} + x\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5.5 \\ 17.5 \end{pmatrix}$ (M1) for valid approach

$8 - x = 5.5$

$x = 2.5$ A1

[2]

(e) $\cos\theta = \frac{(4)(-1) + (5)(3)}{(\sqrt{4^2 + 5^2})(\sqrt{(-1)^2 + 3^2})}$ M1A1

$\cos\theta = 0.5432512782$

$\theta = 0.9964914966 \text{ rad}$

Thus, the required angle is 0.996 rad. A1

[3]

(f) $10 + 3t = 31$ (M1) for valid approach

$3t = 21$

$t = 7$ (A1) for correct value

The amount of time needed

$= 7 + 2$

$= 9 \text{ s}$ A1

[3]

5. (a) $0 \le y < 6$ A1

[1]

(b) $\det(\mathbf{M} - \lambda \mathbf{I})$

$= \begin{vmatrix} -6-\lambda & 0 \\ -1 & 5-\lambda \end{vmatrix}$ (M1) for valid approach

$= (-6-\lambda)(5-\lambda) - (0)(-1)$

$= -30 + 6\lambda - 5\lambda + \lambda^2$

$= \lambda^2 + \lambda - 30$ A1

[2]

(c) $\lambda_1 = -6$, $\lambda_2 = 5$ A2

[2]

(d) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ A2

[2]

(e) (i) $\mathbf{X} = Ae^{-6t} \begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix} + Be^{5t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} 22 \\ 5 \end{pmatrix} = Ae^{-6(0)} \begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix} + Be^{5(0)} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (M1) for substitution

$\begin{cases} 22 = A \\ 5 = \frac{1}{11} A + B \end{cases}$

By solving this system, $A = 22$ and $B = 3$. (A1) for correct values

$\therefore x = 22e^{-6t}$ A1

(ii) $y = 2e^{-6t} + 3e^{5t}$ A1

[5]

(f) (i) The population of brown bear will approach zero. A1

(ii) The population of giant panda will increase exponentially. A1

[2]

6. (a) V_2

$= V - V_1$

$= 29\sin(6\pi t - 0.31) - 23\sin(6\pi t - 0.17)$

$= \text{Im}(29e^{(6\pi t-0.31)\text{i}}) - \text{Im}(23e^{(6\pi t-0.17)\text{i}})$ (M1) for valid approach

$= \text{Im}(29e^{(6\pi t-0.31)\text{i}} - 23e^{(6\pi t-0.17)\text{i}})$ (A1) for correct approach

$= \text{Im}(e^{6\pi t\text{i}}(29e^{-0.31\text{i}} - 23e^{-0.17\text{i}}))$

$\therefore z - w = 29e^{-0.31\text{i}} - 23e^{-0.17\text{i}}$ A1

[3]

(b) (i) $z = 29e^{-0.31\text{i}}$

$z = 29(\cos(-0.31) + \text{i}\sin(-0.31))$ A1

(ii) $w = 23e^{-0.17\text{i}}$

$w = 23(\cos(-0.17) + \text{i}\sin(-0.17))$ A1

[2]

(c) (i) $z - w$

$= 29(\cos(-0.31) + \text{i}\sin(-0.31))$

$-23(\cos(-0.17) + \text{i}\sin(-0.17))$

$= (29\cos(-0.31) - 23\cos(-0.17))$

$+\text{i}(29\sin(-0.31) - 23\sin(-0.17))$ (M1) for valid approach

$= 4.949223888 - 4.955506428\text{i}$ (A1) for correct values

L

$= \sqrt{4.949223888^2 + (-4.955506428)^2}$ M1

$= 7.003703381$

$= 7.00$ A1

(ii) α

$= \tan^{-1}\frac{-4.955506428}{4.949223888}$ M1

$= -0.7860324602$

$= -0.786$ A1

[6]

(d) V_2

$= \text{Im}(e^{6\pi t\text{i}}(z - w))$

$= \text{Im}(e^{6\pi t\text{i}} \cdot 7.003703381e^{-0.7860324602\text{i}})$ (M1) for substitution

$= \text{Im}(7.003703381e^{6\pi t\text{i} - 0.7860324602\text{i}})$ (A1) for correct approach

$= 7.003703381\sin(6\pi t - 0.7860324602)$

$= 7.00\sin(6\pi t - 0.786)$ A1

[3]

7. (a) (i) 5 A1

(ii) 4 A1

(iii) 4 A1

(iv) $43 A1

(v) $61 A1

[5]

(b) For any four edges correct A1
For any eight edges correct A1

1. Choose HA of weight 12
2. Choose AB of weight 22
3. Choose BC of weight 14
4. Choose CD of weight 15
5. Choose DE of weight 16
6. Choose EF of weight 18
7. Choose FG of weight 24
8. Choose GH of weight 17
9. Choose HE of weight 20
10. Choose EA of weight 25
11. Choose AE of weight 25
12. Choose EB of weight 10

Thus, a possible route contains HA, AB, BC, CD, DE, EF, FG, GH, HE, EA, AE and EB. A1

[3]

(c) $218 A1

[1]

(d) (i) For any five edges correct A1

For any ten edges correct A1

1. Choose BC of weight 14
2. Choose CD of weight 15
3. Choose DE of weight 16
4. Choose EF of weight 18
5. Choose FG of weight 24
6. Choose GH of weight 17
7. Choose HA of weight 12
8. Choose AB of weight 22
9. Choose BE of weight 10
10. Choose EH of weight 20
11. Choose HA of weight 12
12. Choose AE of weight 25
13. Choose EB of weight 10

Thus, a possible route contains BC, CD, DE, EF, FG, GH, HA, AB, BE, EH, HA, AE and EB. A1

(ii) $215 A1

[4]

AI HL Practice Set 4 Paper 3 Solution

1. (a) (i) $26\ \text{km}^2$ A1

(ii) $\frac{1}{3}$ A1

(iii) The required equation:

$$y-6=\frac{1}{3}(x-4)$$ (M1) for substitution

$$y-6=\frac{1}{3}x-\frac{4}{3}$$

$$y=\frac{1}{3}x+\frac{14}{3}$$ A1

(iv) Every position in the Voronoi cell of R_3 has R_3 to be the nearest reservoir. A1

[5]

(b) OF A1

[1]

(c) (i) 14 A1

(ii) 8 A1

(iii) 2 A1

(iv)

$$\mathbf{M}=\begin{pmatrix} 0&1&0&0&1&0&1&0&0&0\\ 1&0&0&0&0&1&0&0&1&0\\ 0&0&0&1&0&1&0&0&1&0\\ 0&0&1&0&0&0&0&1&0&1\\ 1&0&0&0&0&0&0&1&0&1\\ 0&1&1&0&0&0&1&0&0&0\\ 1&0&0&0&0&1&0&1&0&0\\ 0&0&0&1&1&0&1&0&0&0\\ 0&1&1&0&0&0&0&0&0&0\\ 0&0&0&1&1&0&0&0&0&0 \end{pmatrix}$$ A5

[8]

(d)	116		A2	
				[2]
(e)	(i)	BEFGCIDOAHB	A2	
	(ii)	HBCIDOAEFG	A2	
	(iii)	There exists at least one vertex of odd degree.	R1	
				[5]
(f)	(i)	7.66	A1	
	(ii)	8.82	A1	
				[2]

(g) For any three edges correct — A1

For all edges correct — A1

1. Choose AE of distance 5.66
2. Choose EF of distance 2
3. Choose FG of distance 3.16
4. Choose GD of distance 3
5. Choose DO of distance 7
6. Choose OA of distance 10

Thus, the required upper bound is 30.8 km. — A1

[3]

(h) For any two edges correct — A1

For all edges correct — A1

1. Choose EF of distance 2
2. Choose GD of distance 3
3. Choose FG of distance 3.16
4. Choose OF of distance 5.66

Therefore, the distance of a minimum spanning tree after deleting the vertex A is 13.8 km. — A1

The required lower bound

$=13.8+7.66+5.66$

$=27.1\text{ km}$ — A1

[4]

2. (a) (i) 340 g — A1

(ii) $22\ \text{g}^2$ — A1

(iii) $\text{P}(321 < A_1 + O_1 + O_2 < 337)$

$= 0.2611900446$ — (A1) for correct value

$= 0.261$ — A1

[4]

(b) (i) 25 g — A1

(ii) $\sqrt{94}$ g — A2

(iii) $\text{P}(D < 0)$

$= 0.0049607822$ — (A1) for correct value

$= 0.00496$ — A1

[5]

(c) (i) $H_0: \ \mu = 120$ — A1

(ii) $H_1: \ \mu < 120$ — A1

(iii) $p\text{-value} = 0.0339445194$ — (A1) for correct value

$p\text{-value} = 0.0339$ — A1

(iv) The null hypothesis is rejected. — A1

As $p\text{-value} < 0.05$. — R1

[6]

(d) The required probability

$= \text{P}(\text{Reject } H_0 \mid \mu = 120)$ — (M1) for valid approach

$= 0.0672405185$

$= 0.0672$ — A1

[2]

(e) The required probability

$= \text{P}(\text{Not reject } H_0 \mid \mu = 119.6)$ — (M1) for valid approach

$= 0.7728699518$

$= 0.773$ — A1

[2]

(f)	(i)	$v=\sqrt{\frac{6}{n}}$	A1
	(ii)	$2(1.6449v)\le 1.1$	M1A1
		$\therefore 2(1.6449)\sqrt{\frac{6}{n}}\le 1.1$	
		$3.2898\sqrt{\frac{6}{n}}-1.1\le 0$	A1
		By considering the graph of	
		$y=3.2898\sqrt{\frac{6}{n}}-1.1$, $n\ge 53.666698$.	(A1) for correct value
		Thus, the least value of n is 54.	A1

[6]